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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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THE ORADELL DAM OF THE HACKENSACK WATER COMPANY

By Nicholas S. Hill, Jr.,* M. Am. Soc. C. E.

To Be Presented November 4, 1925.

Synopsis

This paper describes a unique type of dam of considerable size designed by the writer in 1911 and built by the Hackensack Water Company in 1920-22. The unusual design was the outgrowth of an attempt to plan a dam to meet special conditions found at a particular site. It was not intended to develop a type of dam which would be applicable to conditions likely to be found frequently, nor is it suggested that the type described offers a more economical substitute for the usual types of dam under ordinary conditions.

The special conditions for which, in combination, this design was intended to provide were:

1.—Spillway crest with maximum height about 20 ft. above the stream bed.
2.—Maximum possible length of dam between abutments, 402½ ft.
3.—Flood flows which would produce a head of 3 ft. over any practicable length of weir. (402½ ft.)
4.—Firm foundation of hardpan and shale 15 to 45 ft. below the stream bed.
5.—Porous sand and gravel overlying the deep firm foundation, with intercalated and lenticular masses of stiff dense clay over part of the section.
6.—Impracticability, because of cost, of adopting a masonry section of the gravity type extending to the deep underlying hardpan and shale.
7.—Impracticability of allowing a head of more than 1 ft. over the spillway crest, because of flooding of railroad and other unowned lands.
8.—Necessity of providing a dam section which might be overtopped without damage in times of flood.
9.—Necessity of holding foundation and toe pressures within lowest possible limits.
10.—Necessity of providing solid cut-off all the way down to the underlying hardpan and shale.

Introduction

The Hackensack Water Company, chartered in 1869, is one of the largest private water companies in the United States. At the time the Oradell Dam

Note.—This paper is issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on the paper will be closed in **February, 1926.** and, when finally closed, the paper, with discussion in full, will be published in *Transactions.*

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was constructed it supplied water to forty-eight municipalities or civil divisions in Hudson and Bergen Counties, New Jersey, all lying within the Metropolitan Area of New York, N. Y. The most populous of these communities were Hoboken, West Hoboken, West New York, North Bergen, Weehawken, and the Town of Union, in Hudson County, and Hackensack, Englewood, Rutherford, and Ridgefield Park, in Bergen County.

According to the 1920 Census, the total population in the territory supplied was approximately 340 000, of which about 92% was connected to the mains of the Company. It is estimated that this population will have increased to 530 000 by 1940.

The daily volume of water delivered for distribution increased from about 26 000 000 gal. in 1912 to from 35 000 000 to 36 000 000 gal. in 1918, 1919, and 1920. It is estimated that the water delivered for distribution will have increased to more than 80 000 000 gal. daily by 1940. From 40 to 45% of the consumption is, and probably will remain, industrial in character.

The supply is taken from the Hackensack River at New Milford, N. J., just above tide-water, at which point the stream has a drainage area of 115 sq. miles. At New Milford the entire supply is filtered through mechanical gravity filters and pumped for distribution at heads of 300 and 600 ft.

At present, the Water Company has two impounding reservoirs on the water-shed above New Milford to regulate the flow and augment the supply during dry weather. The necessity for impounding storage is indicated by the fact that the natural flow of the Hackensack River at the New Milford intake decreases to 10 000 000 gal. per day or less in extremely dry seasons. The safe yield of the stream with the present storage (about 3 900 000 000 gal.) is estimated to be 40 000 000 gal. per day. Prior to the construction of the new Oradell Dam, the safe yield was only 32 000 000 gal. per day, and the impounding storage 2 190 000 000 gal.

The new Oradell Dam creates a storage reservoir of 3 023 000 000 gal. capacity, which in conjunction with the smaller Woodcliff Reservoir of 889 000 000 gal. capacity, constructed in 1906, comprises the total impounding storage of the Water Company on the Hackensack River. The new dam is situated on the main stream just above Oradell, N. J., from which it takes its name. The site is only about ¾ mile up stream from the New Milford pumping station and intake.

Aside from the much smaller Saddle River, the basin of which adjoins that of the Hackensack on the west, the Hackensack River is the only readily available source of water supply for the large and rapidly growing population served by the Hackensack Water Company. With proper storage it is estimated that the Hackensack River may be made to supply the needs of this territory until 1940, when the population supplied will exceed 500 000.

Since 1903, the Company has maintained a dam on the present site at Oradell, which originally was an old mill dam of low elevation (spillway crest, 8.60 ft.*) and negligible storage. The basin above the dam was a swamp quite heavily timbered. Beginning in the spring of 1911, the work

* All elevations given in this paper refer to Atlantic mean high tide.

of creating a storage reservoir at Oradell, which had been started in 1903 and then abandoned in favor of the Woodcliff site, was resumed. The clearing of the basin was again undertaken and a temporary timber crib dam, with wooden sheet-pile cut-off, was constructed on the site of the old mill dam. The crest of this crib dam which was about 11 ft. above the stream bed (or Elevation 11), created a storage basin of sufficient depth to float a dredge. Two suction dredges, one 12 in. and the other 20 in., were constructed on the bank of this small basin, and launched. Between 1911 and 1920, the basin, up stream from the dam, and below the 11-ft. flow line, was dredged to Elevation 0, the excavated material being used to build up the banks and fill in depressions, to which it was pumped through pipe lines supported on floats.

Coincidentally with the starting of this work, borings were made across the valley of the Hackensack River below the temporary crib dam, to determine the best site for a permanent dam to create a reservoir having a flow line 20 ft. above the stream bed. As a result of these borings the location for the permanent dam was chosen on a line at right angles to the thread of the stream and 50 to 90 ft. down stream from the curved crib dam. The permanent dam was designed in 1911 and its construction was begun in 1920, at which time the dredging was practically complete. By the close of 1921 the spillway section had been built to Elevation 16. The dam was completed to Elevation 20 in 1922.

Considerations Controlling the Selection of the Oradell Reservoir Site

The primary consideration controlling the selection of the Oradell Reservoir site was the desire to obtain a considerable volume of storage in the lower reaches of the drainage basin, near the intake, at a point so located that all branches of considerable size would be tributary to it, thus insuring the highest possible degree of stream regulation and control over flood flows.

The selection of any site other than that at Oradell would have excluded from the drainage basin tributary to the reservoir, an area of at least 25 sq. miles, or 22% of the total drainage area. The run-off from this area would have been ineffective in replenishing any other reservoir, following depletion, and the flood flows from its 25 sq. miles entirely uncontrolled except in so far as regulation could be effected by the method of operating the suction basin and pumping station at New Milford.

Studies indicated that the cost per 1 000 000 gal. of developing storage in Oradell Reservoir would be no higher than at any other desirable site.

Considerations Controlling the Selection of the Flow-Line Elevation of Oradell Reservoir

The divide between the Hackensack drainage and that of the Sparkill, a tributary of the Hudson, has an elevation only about 30 ft. above mean sea level, or 27.5 ft. above the Company's datum which is mean high tide. The two-track New Jersey and New York Railroad at Oradell is about at Elevation

23. Surveys of the reservoir basin below Contour 25, which were started in 1910, indicated that the placing of the spillway crest above Elevation 20 would necessitate the relocation of railroads and highways and the purchase of land areas the cost of which would have been greater than warranted by the advantages obtained. It was decided, therefore, after due study and consideration, to fix the spillway crest at Elevation 20 and to provide in the dam a sluice-gate section of a capacity sufficient to pass all extremes of flood with a head not exceeding 1 ft. over the spillway crest. In this way it is expected that troubles due to back-water and flooding above the dam will be avoided.

Flood Flows and Proportions of Sluice-Gate and Spillway Sections

The severity of the floods which occur on the Hackensack River is not extreme when measured in second-feet per square mile. Considerable trouble from flooding, however, has been experienced at times of extremely high run-off because of channel obstructions below the dam at Oradell. There are islands in the stream between Oradell and the pumping station at New Milford. The New Milford pumping station is located on an island, the surface of which is at a general elevation of 10 ft.

The suction basin has been created by damming the various arms of the stream between islands and the mainland in the vicinity of the pumping station. The crests of these dams stand at Elevation 4.93, so that under normal conditions of stream flow the flow line in the suction basin and stream between Oradell and New Milford stands at about 5 ft.

On one occasion, during a period of very high run-off, the water surrounding the New Milford pumping station rose for a short time to a height above the engine-room floor, which stands at Elevation 10. Following this experience the suction basin and stream channels between Oradell and New Milford were considerably enlarged and improved by dredging. This work was done about 1911 or 1912.

As a result of the removal of obstructions and other improvements accomplished at this time, it is believed that the water level below the New Milford pumping station cannot now rise to an elevation exceeding 8 ft., and that the maximum elevation of back-water at the toe of the Oradell Dam, $\frac{3}{4}$ mile above the pumping station, cannot in the future exceed Elevation 12.25. The minimum net head on the sluice-gates in the Oradell Dam, with 1 ft. over the spillway crest, will, therefore, not be less than $21.00 - 12.25$, or 8.75 ft., at times of most extreme flood.

It is believed that the most extreme flood likely to occur on the Hackensack River will not exceed 46 000 000 gal. daily per sq. mile of tributary drainage area. Unfortunately, the run-off data on the Hackensack, which are meager, do not afford a basis for determining the extent of the floods which have occurred in the past. This figure, therefore, is an estimate. As a basis for the estimate, a study was made of the recorded flood flows on many streams of similar size in New Jersey and New York, some of which are given in Table 1.

TABLE 1.

Name of stream.	Area of water-shed, in square miles.	Maximum recorded flood flow, in millions of gallons per day.	Date of occurrence.
Pequannock, N. J.	85.0	41.8	Feb. 6, 1896
Ramapo, N. J.	86.0	36.2	"
Wanaque, N. J.	101.0	46.1	"
Rockaway, N. J.	118.0	29.8	"
Paulins Kill, N. J.	126.0	34.9	"
Musconetcong, N. J.	130.0	29.1	"
Ramapo, N. J.	160.0	42.7	Sept. 22, 1882
Esopus Creek, N. Y.	234.0	43.6	Feb. 15, 1908

With these figures, a comparative study was made of the Hackensack River and the other streams mentioned, and in almost every case it was found that the drainage basins of the other streams were steeper and had a more rapid run-off than that of the Hackensack.

The length of spillway and the number of sluice-gates were proportioned, therefore, to pass a flood of 46 000 000 gal. daily per sq. mile, equivalent to 71 sec-ft. per sq. mile, or a total of 8 094 sec-ft., with 1-ft. head on the spillway crest, this flood to be passed when the back-water against the down-stream face of the dam stands only 7.75 ft. below the spillway crest elevation of 20 ft.

In connection with these assumptions as to the maximum probable flood flows, it may be noted that a flood in excess of the assumed maximum will not endanger the stability of the dam, which has a sufficient margin of safety to permit a head of 3 ft. on the spillway crest. The top of the sluice-gate section and the abutment and wing-walls were also placed 3 ft. above the spillway crest. With this head on the crest, and back-water on the down-stream face at 7.75 ft. below a spillway crest, the dam will pass a flood of 75 000 000 gal. per day per sq. mile, which is equivalent to 4⅛ in. of depth on the drainage area in 24 hours. It is not believed that such a flood can occur, but if it should, it will result in nothing more serious than the temporary flooding of certain railroad and unowned lands above the dam.

Stratigraphy at the Dam Site

In June and July, 1911, thirty-one test borings, ranging in depth from 20 to 45 ft., were made in the vicinity of the proposed dam.

The contract for the boring called for thirty-one 3-in. borings, by the auger, hydraulic, and drive-well process, at points designated by the writer. Casings, 3 in. in diameter, were driven to impenetrable hard bottom, of hard-pan, boulder, or rock, and samples furnished for every 2 ft. of depth and at changes in the stratification. The price for this work was $3 per ft. In addition, diamond drill core borings were made, as directed, in test wells which had been driven to refusal, and core samples furnished. The diamond drill core borings were made with a ⅛-in. diamond core bit, at a cost of $8 per ft.

The contract also called for auger borings to be made wherever samples could be obtained. When material could not be brought up with the auger

a sand bucket was used. Wash borings were made only when the auger and sand-bucket methods failed.

The location of the borings, and the stratigraphy disclosed by part of the borings are shown on Fig. 1. The outlines of the dam shown in this view are those of the original mill dam previously mentioned, the crest elevation of which was 8.6 ft. Two lines of borings, 15 ft. apart, with staggered holes, were made entirely across the valley just below the old dam. This was regarded as the most desirable location for the new dam because of the surface topography. A third line of borings was made across the valley about 375 ft. down stream from the first two lines, and these parallel lines were connected by a series of borings which approximately paralleled the thread of the stream.

The nature of the foundation as disclosed by the borings was disturbing. The hardpan or shale which lay at a depth of only 10 to 15 ft. below mean high tide at the extreme westerly end of the sections, fell off to a depth of 45 ft. below mean high tide at the easterly end.

A masonry dam of the gravity type might have been extended to this foundation, but its cost would have been prohibitive. The overlying materials were pervious and of unsatisfactory bearing value. An earth dam with a sheet-pile cut-off, and a large sluice-gate section for passing flood flows, would have served, but it was considered unwise to rely solely upon sluice-gates for the discharge of floods, and an earth dam if overtopped would have been destroyed.

The purpose of the design adopted was to retain the advantages of an earth dam with sluice-gate section and sheet-pile cut-off, and, at the same time, insure it against damage from overtopping the crest.

Design of Sluice-Gate Section

No difficulty was presented by the sluice-gate section, which was located at the extreme westerly end of the dam, opposite the main channel, where the underlying hardpan or shale lay reasonably near the surface. The sluice-gate section, therefore, was carried down to a secure underlying foundation at all points.

The sluice-gate section, from the west abutment to the wall which separates it from the spillway section, was divided into seven equal bays by concrete piers (as shown by the general plan, Fig. 2, and the horizontal and vertical section, Fig. 3). Each pier was founded on the underlying hardpan. The space between the piers was floored over at Elevation 0 with a reinforced concrete slab, and a concrete cut-off wall extending from end to end of the sluice-gate section was extended from the under side of the floor to the hardpan, thus affording a solid masonry cut-off the full length of the sluice-gate section. Above the floor on the line of the cut-off wall were placed the seven rectangular sluice-gates, each 7 by 9 ft. in size, one in each bay. The sills of the gates were placed at Elevation 0. Each gate is operated manually from a stand seated on the concrete platform or walkway which extends from end to end of the sluice-gate section spanning the tops of the piers which separate the bays.

Under normal conditions all water passing the dam will be passed through the sluice-gates. No water will be allowed to flow over the spillway except

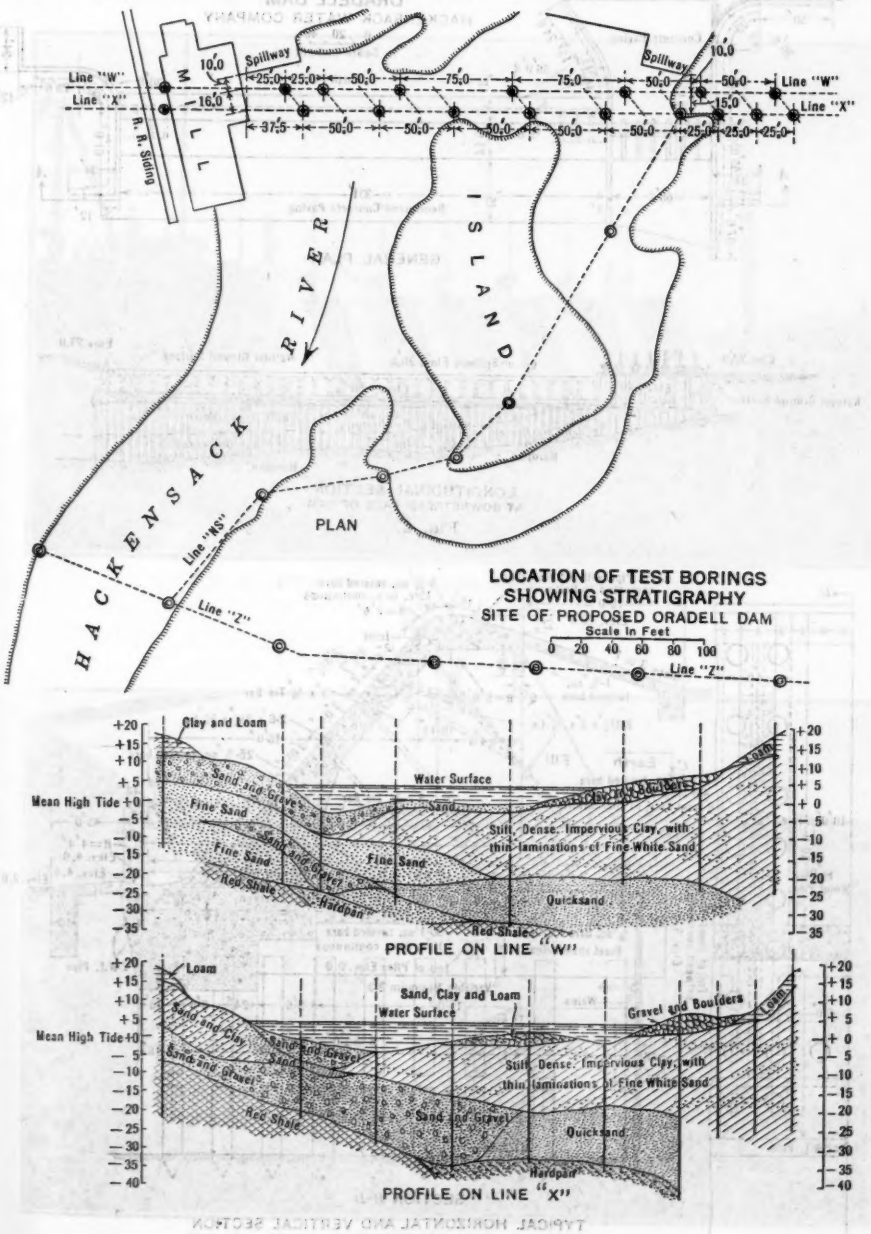


FIG. 1.

during the most extreme flood. Views of the sluice-gate section are shown in Figs. 4 and 5.

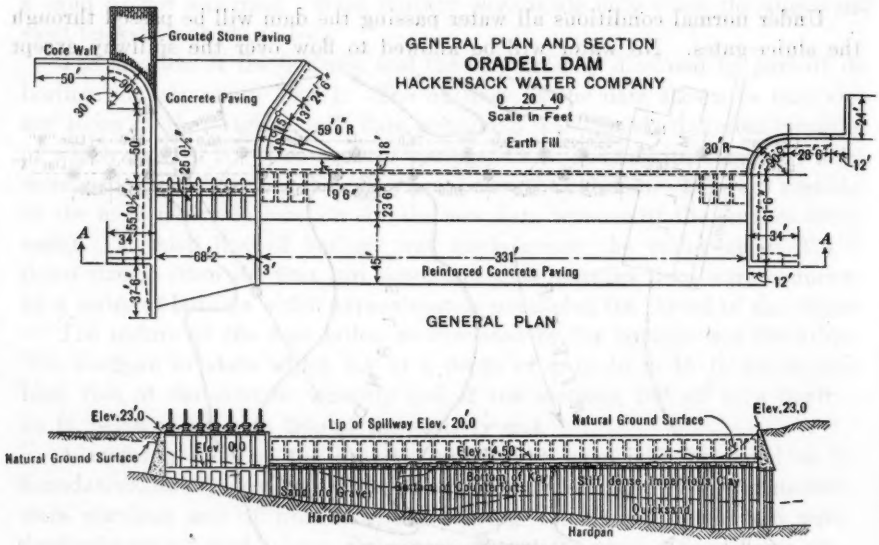

GENERAL PLAN AND SECTION
ORADELL DAM
HACKENSACK WATER COMPANY
0 20 40
Scale in Feet
Core Wall
Grouted Stone Paving
Concrete Paving
Earth Fill
Reinforced Concrete Paving
GENERAL PLAN
Elev. 23.0
Natural Ground Surface
Lip of Spillway Elev. 20.0
Elev. 0.0
Elev. -5.0
Bottom of Key
Bottom of Counterforts
Sand and Gravel
Hardpan
Stiff dense Impervious Clay
Quicksand
LONGITUDINAL SECTION
AT DOWNSTREAM FACE OF DAM

FIG. 2.

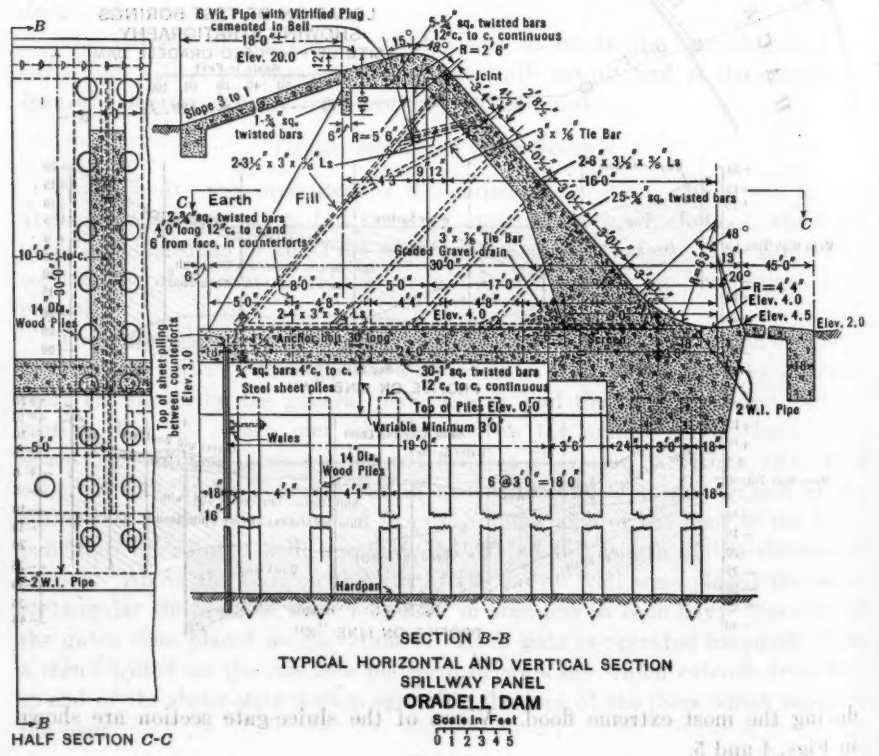

8 Vit. Pipe with Vitrified Plug cemented in Bell
Elev. 20.0
Slope 3 to 1
Earth Fill
Graded Gravel drain
14" Dia. Wood Piles
Top of sheet piling between counterforts Elev. 3.0
Steel sheet piles
Wales
Top of Piles Elev. 0.0
Variable Minimum 3'0"
Hardpan
2 W.I. Pipe
SECTION B-B
HALF SECTION C-C
TYPICAL HORIZONTAL AND VERTICAL SECTION
SPILLWAY PANEL
ORADELL DAM
Scale in Feet
0 1 2 3 4 5

FIG. 3.

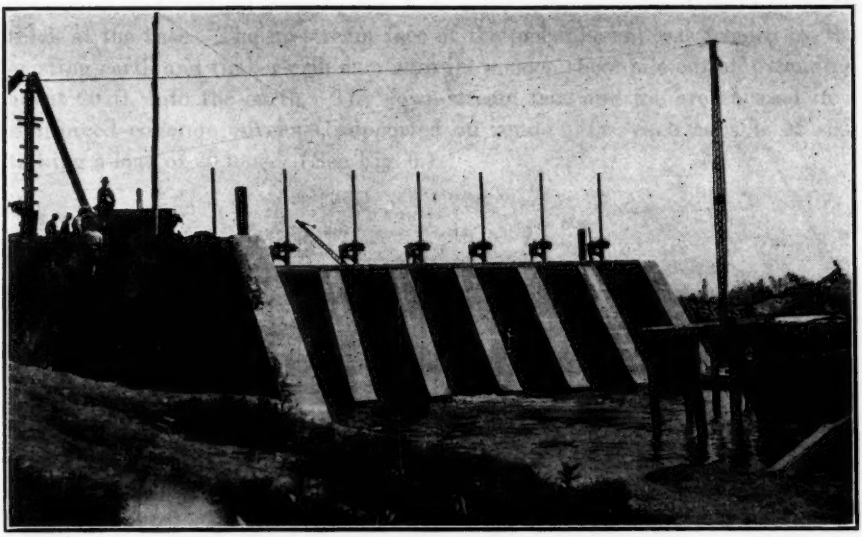


FIG. 4.—VIEW OF SLUICE-GATE SECTION, ORADELL DAM.

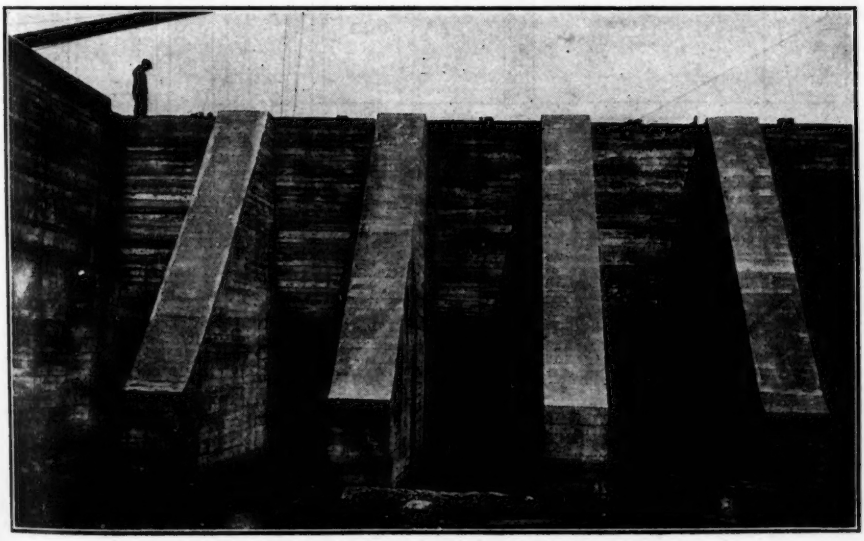
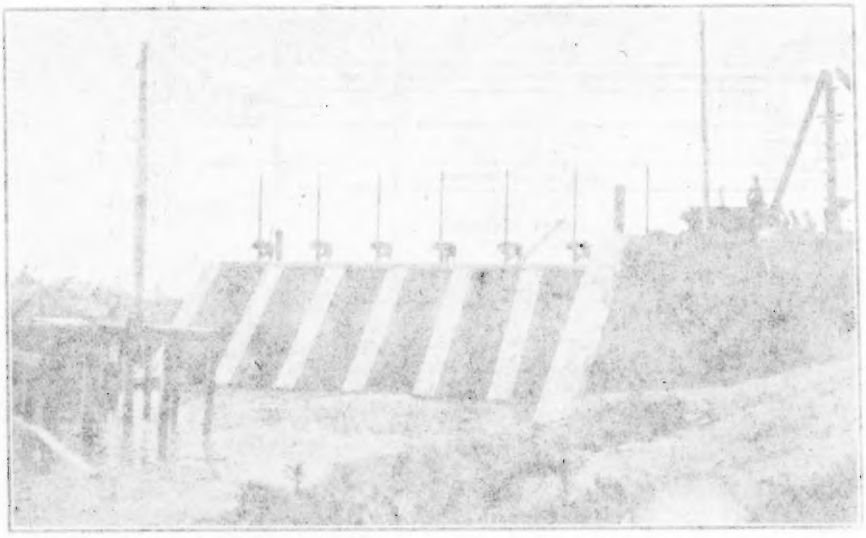
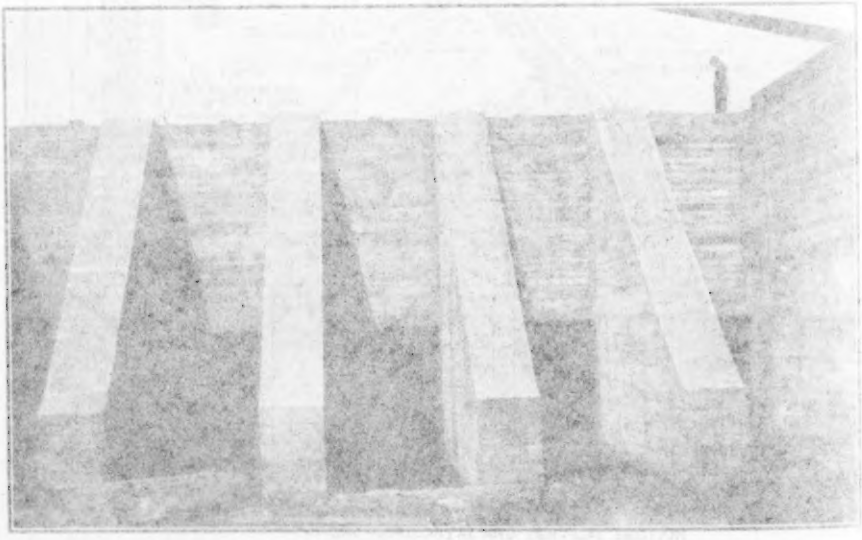


FIG. 5.—VIEW OF SLUICE-GATE SECTION, ORADELL DAM.





Design of Spillway Section

The spillway section, as may be seen from the general plan, Fig. 2, and the horizontal and vertical section, Fig. 6, is essentially an earth dam 50 to 90 ft. thick at the base. The up-stream face of the embankment was formed by the existing earth and timber crib dam with its wooden sheet-pile cut-off extending about 20 ft. into the earth. The down-stream face and toe are encased in a reinforced concrete envelope, supported on round piles, each capable of sustaining a load of 20 tons. (See Fig. 6.)

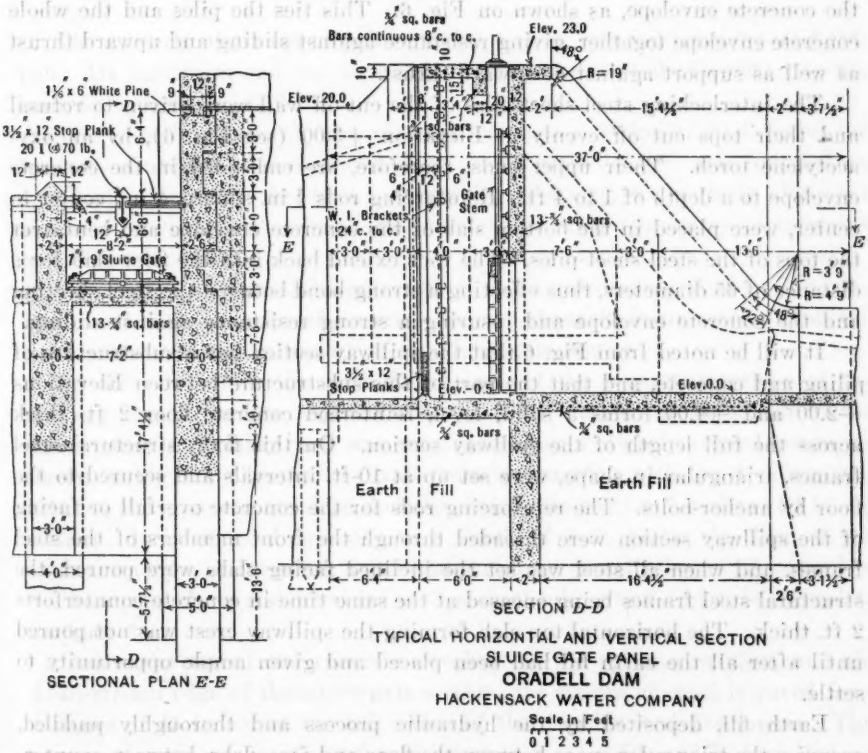


Fig. 6.

At the up-stream edge of the floor of the concrete envelope there is a continuous line of interlocking steel sheet-piling which is also carried far into the easterly bank, following along the face of the easterly concrete abutment. These sheet-piles extend to the hardpan. Their tops are embedded in the floor of the concrete envelope and in the concrete of the easterly abutment wall. Together with the concrete cut-off wall of the sluice-gate section, they form a continuous cut-off entirely across the new dam, approximately on the line of the up-stream edge of the concrete envelope. (See Fig. 6.)

The fill between the existing earth and timber crib dam and the concrete envelope, which forms the body of the new dam, is hydraulically placed and

of stiff clay. This fill forms a stable and impervious dam in itself, except that it is not proof against the inroads of burrowing animals, nor against scouring if it should be overtopped.

The reinforced concrete envelope, supported on round piles, performs the function of a masonry core-wall by resisting the inroads of burrowing animals in the fill above the steel sheet-pile cut-off, and also would protect the top and down-stream face of the fill against scouring if this section of the dam should be overtopped, or because of failure promptly to control the sluice-gates.

The heads of all round piles are embedded from 1 to 2 ft. in the base of the concrete envelope, as shown on Fig. 6. This ties the piles and the whole concrete envelope together, giving resistance against sliding and upward thrust as well as support against downward thrust.

The interlocking steel sheet-piles of the cut-off wall were driven to refusal and their tops cut off evenly at Elevation +3.00 (see Fig. 6), by an oxy-acetylene torch. Their upper ends, therefore, are embedded in the concrete envelope to a depth of 1 to 4 ft. Reinforcing rods $\frac{3}{4}$ in. square, 4 in., center to center, were placed in the bottom slab of the concrete envelope and bent over the tops of the steel sheet-piles. The rods extend back into the floor slab for a distance of 65 diameters, thus effecting a strong bond between the steel sheeting and the concrete envelope and insuring a strong resistance against sliding.

It will be noted from Fig. 6 that the spillway section has a substructure of piling and concrete, and that the part of this substructure between Elevations +2.00 and +4.00 forms a solid, level, reinforced concrete floor 2 ft. thick across the full length of the spillway section. On this floor, structural steel frames, triangular in shape, were set up at 10-ft. intervals and secured to the floor by anchor-bolts. The reinforcing rods for the concrete overfall or facing of the spillway section were threaded through the front members of the steel frames, and when all steel was set the inclined facing slabs were poured, the structural steel frames being encased at the same time in concrete counterforts 2 ft. thick. The horizontal top slab forming the spillway crest was not poured until after all the earth fill had been placed and given ample opportunity to settle.

Earth fill, deposited by the hydraulic process and thoroughly puddled, occupies the triangular space between the floor and face slabs, between counterforts, and extends back to the top of the existing timber crib dam. The sloping top surface of this fill is paved for a distance of 15 ft. back from the up-stream edge of the spillway crest.

In each panel between counterforts of the spillway section a pair of 2-in. weep-holes were placed through the face slab of the concrete envelope at the floor level. The upper ends of these 2-in. pipes were screened with fine copper mesh, and the acute angle between the floor and face slabs of the envelope was then filled with a triangular fillet of graded gravel and sand, which was intended to facilitate the drainage and consolidation of the fill and, at the same time, prevent the washing out of the filling material. The weep-holes

should also relieve the face slab from any direct water pressure which might otherwise occur if the fill should settle away slightly from the concrete.

It will be noted that a considerable volume of concrete was saved by the adoption of the type of construction shown, thus lessening the dead weight of the dam, and also that under the extreme conditions of loading later described the downward pressure is considerably reduced (as compared with the pressures which would have obtained with a solid masonry section) by the action of the upward water pressure on the under side of the face slab.

Other Features of Design

The westerly abutment of solid gravity section was carried down to hardpan. Its up-stream end was curved on a 30-ft. radius and extended into the bank for a distance of 80 ft.

The easterly abutment, also of solid masonry and similar in form to the westerly abutment, was extended down to Elevation —3.00 and founded on round piles driven to hardpan. As previously stated, the easterly abutment was also provided with a steel sheet-pile cut-off where necessary to intercept possible under flow.

The approach to the sluice-gates, which has a uniform elevation of —0.5 ft., was paved with a reinforced concrete slab for about 90 ft. up stream from the gates and across the full width of the sluice-gate section.

In order to retain the earth filling back of the concrete envelope of the spillway section and to separate this fill from the channel approach to the sluice-gates, a dividing wall was constructed, on a slightly curved sweep, as shown in Fig. 2. This wall extends from its junction with the dividing wall between the concrete envelope and the sluice-gate section for about 80 ft. up stream. It is of solid masonry to a depth of 3 ft. below the floor of the approach to the sluice-gates, and is supported on round piles which extend to the hardpan. It is also provided with a sheet-pile cut-off which is carried to hardpan.

Below the toe of the concrete envelope of the spillway section, and the down-stream edge of the sluice-gate section, the stream channel is paved for a distance of 45 ft. across the full width of the dam between abutments. This pavement or apron is of reinforced concrete 8 in. thick and is turned down into the earth for a distance of several feet at its down-stream edge.

The concrete envelope was keyed into the face of the easterly abutment wall and into the face of the dividing wall between the sluice-gate and spillway sections, thus forming expansion joints at these points. There are no expansion joints in the 330 ft. of concrete envelope except at the ends, as noted. The concrete envelope has been subjected to a wide range of temperature with full water pressure behind it. The end joints have opened as much as $\frac{1}{8}$ in. No visible cracks have occurred at intermediate points in the envelope and no visible leakage has appeared at any point. There are practically no damp spots visible on the face of the envelope under full water pressure.

The abutment walls, which are unreinforced, were cast in sections with tongue and grooved joints at approximately 30-ft. intervals. Under extreme range of temperature each of these joints has opened about ½ in.

Unit Stresses

Each round pile under the spillway section was driven to sustain safely a load of 20 tons as determined by the so-called *Engineering News* formula for drop-hammer driving. As previously stated, all steel sheet-piles were driven to refusal.

All concrete was mixed in proportions of 1 : 2.5 : 5, and all reinforced concrete was designed on the basis of the following units:

$K = 0.65$	$C = 450$
$p = 0.0052$	$r = 15$
$S = 14\,000$	$E_c = 2\,000\,000$

Construction Methods

No cableway was used in the construction of the dam. Excavated material was handled with derricks. All concrete was mixed in a single mechanical batch mixer situated on the bank nearest which work was being done, and handled to place by derricks and the ordinary steel double-side dump-cars running on portable narrow-gauge track.

A Blaw-Knox steel form and form carrier were used in casting the face slab of the concrete envelope which was poured in 30-ft. sections. The joints were made at the center of the counterforts in order to secure water-tightness. The heavy face reinforcement was carried through these joints. The steel form and form carrier are shown in Fig. 7.

Two types of pile-driver were used on the work. The round and steel sheet-piles across the full length of the spillway section were driven by a drop-hammer weighing 2 950 lb. The other piles were driven with a double-acting steam hammer suspended from a derrick boom. The total weight of this hammer was 3 500 lb., and the weight of the ram was 575 lb. The steam cylinder was 6 by 10 in., and at a steam pressure of 60 lb. per sq. in. the strokes averaged about 170 per min.

Experiments were made on test piles to determine the ratio of penetration per stroke to give the same bearing value, the double-acting steam hammer being compared with the drop-hammer under the particular conditions which obtained on this work. The experiments suggested the following as a desirable general formula applicable to all methods of pile-driving, including drop-hammer, single-acting steam hammer, and double-acting steam hammer:

$$L = \frac{2\,W \times h}{S + \frac{\text{Constant}}{N}}$$

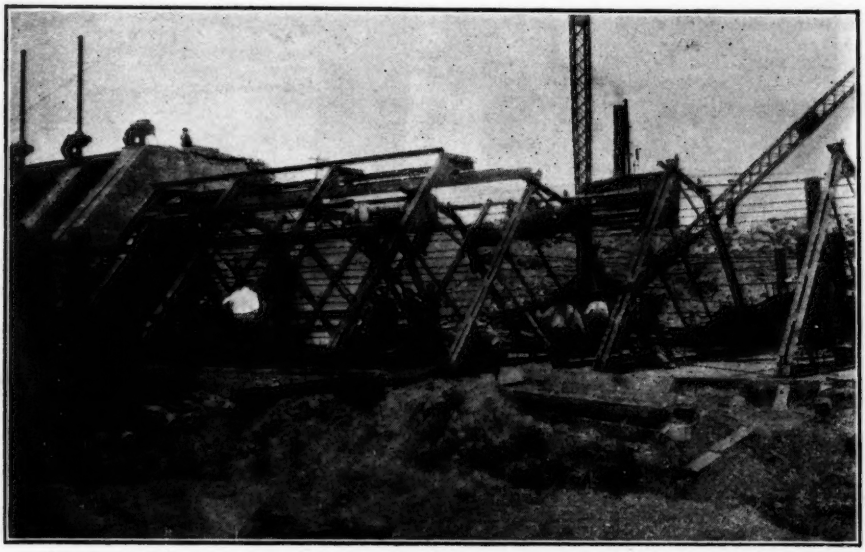


FIG. 7.—STEEL FORM AND FORM CARRIER USED IN CASTING FACE SLAB OF CONCRETE ENVELOPE, ORADELL DAM.

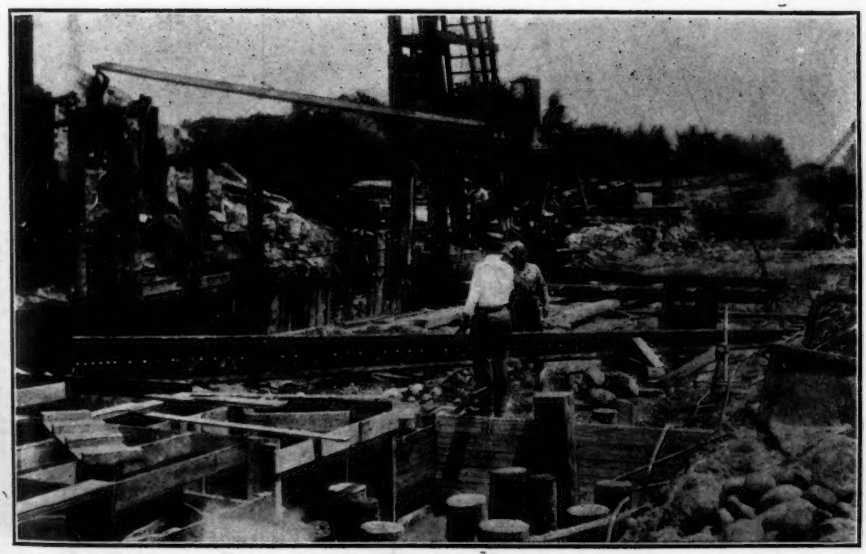
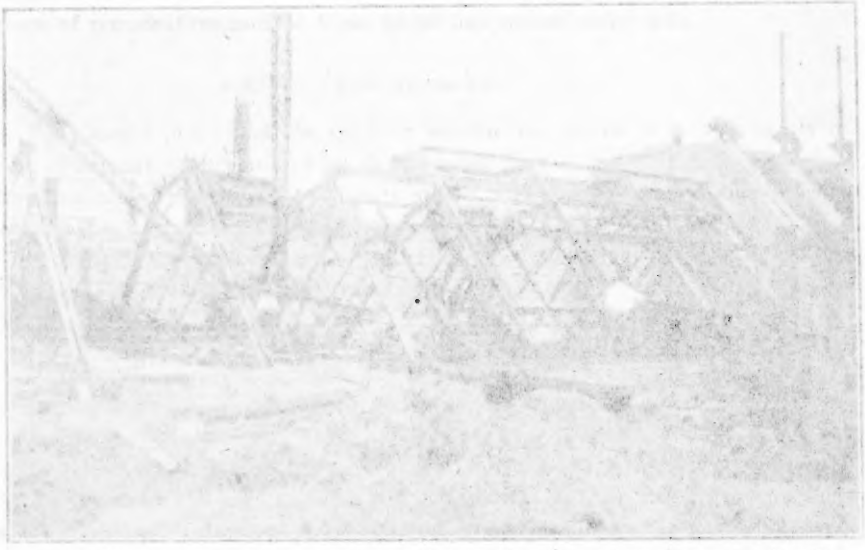


FIG. 8.—WEDGE-PILE USED ON ORADELL DAM.



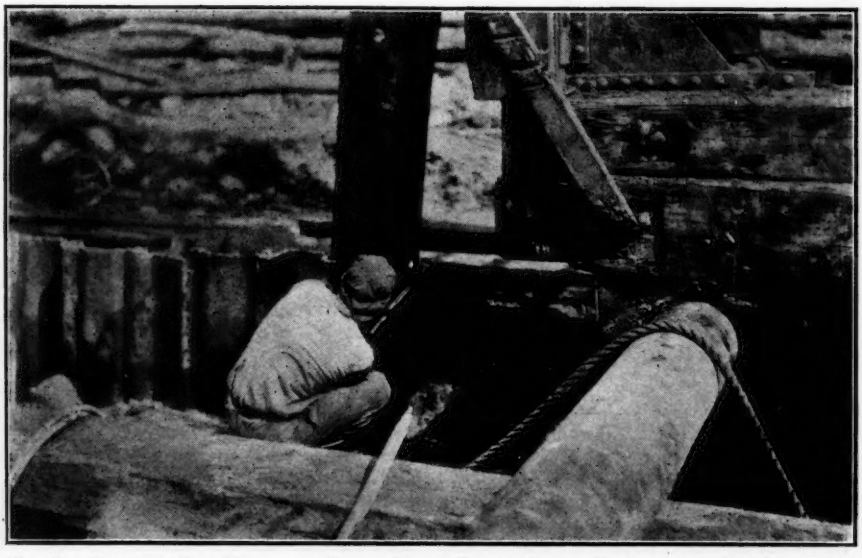


FIG. 9.—CUTTING OFF HEADS OF STEEL SHEET-PILES, AFTER DRIVING, BY OXY-ACETYLENE TORCH, ORADELL DAM.

FIG. 10.—METHOD OF DISTRIBUTION OF HYDRAULIC FILL BACK OF SPILLWAY SECTION, ORADELL DAM.

FIG. 9.—CUTTING OFF HEADS OF STEEL SHEET-PILES AFTER DRIVING BY OXY-ACETYLENE TORCH. ORANGE PARK.

FIG. 10.—METHOD OF DISTRIBUTION OF HYDRAULIC FILL BACK OF SHELLWAY SECTION. ORANGE PARK.

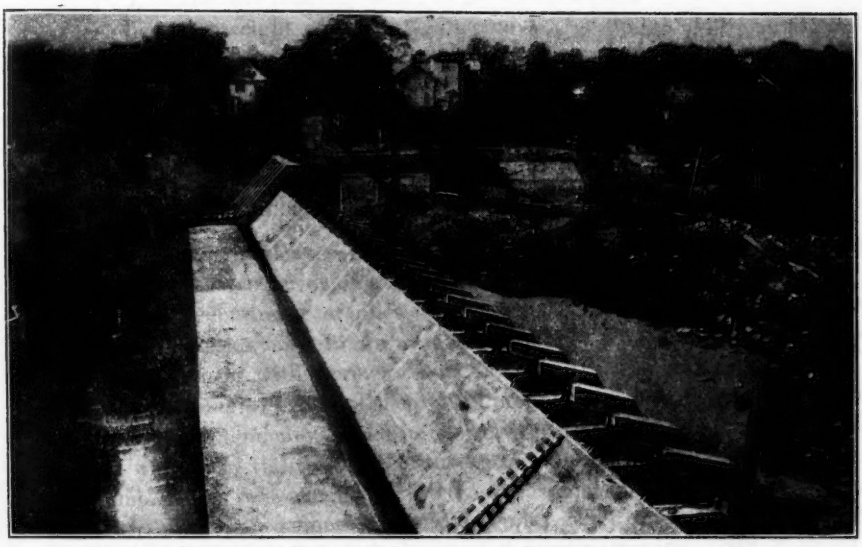


FIG. 11.—VIEW FROM EASTERLY ABUTMENT OF SPILLWAY AND SLUICE-GATE SECTIONS, ORADELL DAM, BEFORE COMPLETION.

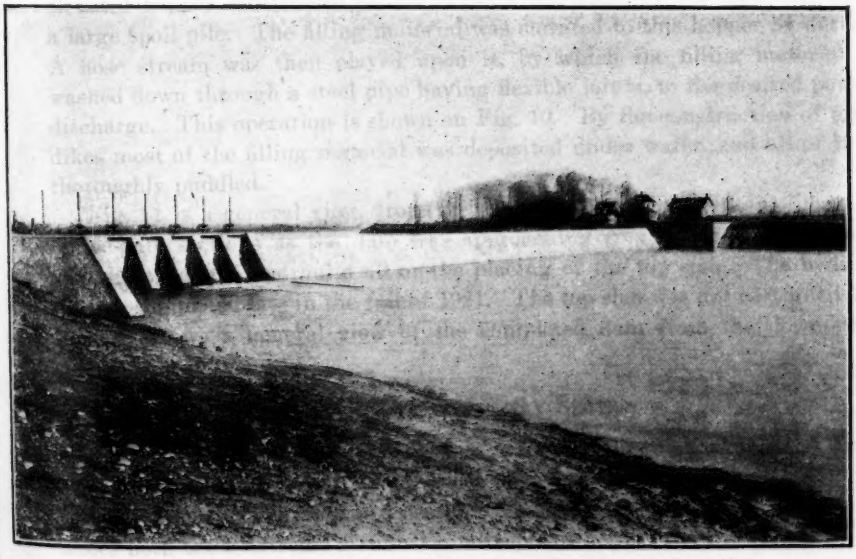


FIG. 12.—VIEW OF COMPLETED ORADELL DAM FROM DOWN-STREAM SIDE.

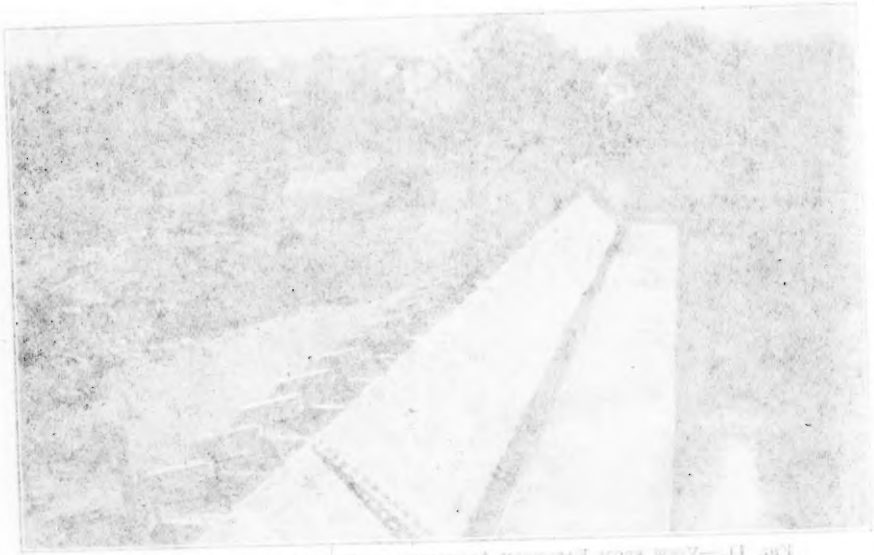


FIG. 11.—VIEW FROM EASTERLY ABUTMENT OF SPILLWAY AND SLUICE-GATE SECTIONS, CROTON DAM, BEFORE COMPLETION.

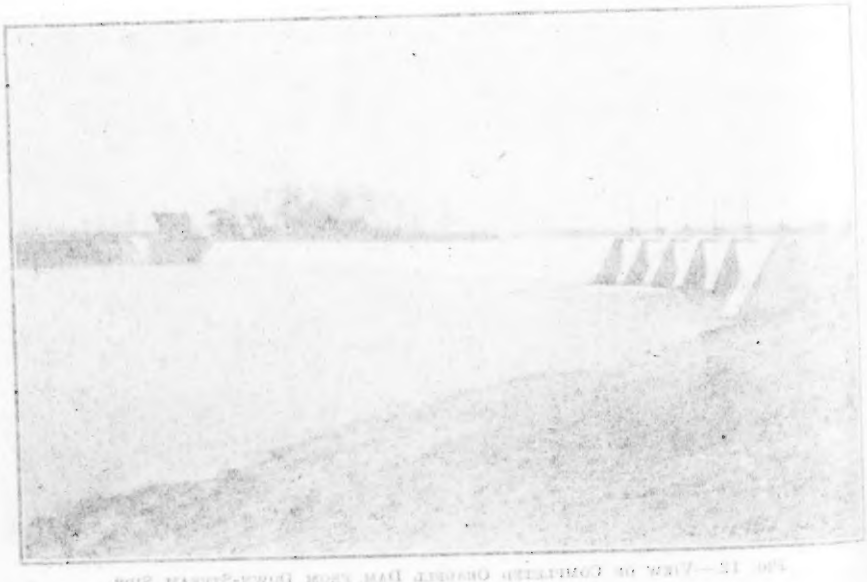


FIG. 12.—VIEW OF COMPLETED CROTON DAM, FROM DOWN-STREAM SIDE.

in which

L = the safe bearing value, in pounds per pile;
W = the weight of the ram, in pounds, for drop-hammers and single-acting steam hammers, and the weight of the ram plus the mean effective steam pressure for double-acting steam hammers;
h = the length of stroke, in feet;
S = the penetration, in inches, under the last blows of the hammer;
N = the number of strokes per minute.

The constant in the second term of the denominator should be determined by a comprehensive series of experiments.

The average number of piles driven per day on the spillway section was 33 for round piles, and 15 for steel sheet-piles. The driving of these piles occupied 2 months with a crew of 8 men.

As the driving of the steel sheet-piles progressed, some difficulty was experienced from the tendency of the joints between the sheet-piles to spread at the bottom while held tightly together at the top, thus causing the piles to get out of plumb. This was corrected by the occasional driving of wedge-piles as required. Fig. 8 shows one of the wedge-piles which was made on the ground by cutting and riveting together two of the ordinary piles. Fig. 9 shows the operation of cutting off the heads of the steel sheet-piles after driving by the oxy-acetylene torch.

The hydraulic fill back of the spillway section was distributed from an elevated hopper constructed on the easterly abutment of the dam adjacent to a large spoil pile. The filling material was elevated to this hopper by derricks. A hose stream was then played upon it, by which the filling material was washed down through a steel pipe having flexible joints, to the desired point of discharge. This operation is shown on Fig. 10. By the construction of proper dikes most of the filling material was deposited under water, and all of it was thoroughly puddled.

Fig. 11 is a general view, from the easterly abutment, of the spillway and sluice-gate sections as the dam was approaching completion, but before the completion of the hydraulic fill or the placing of the top slab. The hydraulic fill was completed late in the fall of 1921. The top slab was not cast until 1922. Fig. 12 shows a general view of the completed dam from the down-stream side.

Stability—Spillway Section

Forces.—Two conditions of loading, subsequently referred to as *A* and *B*, were selected as representing the most extreme conditions which might conceivably occur. The spillway section (concrete envelope) was made stable under both these extreme assumed conditions of loading.

Condition A.—The concrete envelope was assumed to have been constructed to its ultimate height (crest elevation, 20 ft.), and then subjected to a maximum flood overtopping its spillway crest by 1 ft. before any of the earth fill

had been made. This condition could have occurred only during the construction period and only in event the dam had been built to its full height at the first operation. As actually constructed (concrete envelope and earth fill carried to Elevation 16 and raised later to Elevation 20), the nearest approach to Condition *A* which can be imagined would be that in which, after construction to the full height, the earth fill should settle away slightly from the inner face of the concrete slab, at a time when water was flowing over the spillway, thus causing full water pressure against the slab; but as constructed this could not occur without, at the same time, having the benefit of the weight of a considerable portion of the earth fill on the floor of the envelope in resisting overturning.

Condition B.—The same conditions as to loading as Condition *A* were assumed except that the resisting moment and downward pressure due to the weight of earth on the floor of the envelope was also taken into account.

With one exception which represented a most extreme condition not possible to be found save under very exceptional circumstances during the construction period, the four resultants tested cut the base practically at the mid-point.

MOMENTS IN RESTRAINED AND CONTINUOUS BEAMS BY THE METHOD OF CONJUGATE POINTS*

By L. H. Nishkian† and D. B. Steinman,‡ Members Am. Soc. C. E.

Synopsis

The object of this paper is to present a rapid and direct graphic solution of the relation expressed by the Theorem of Three Moments, and to indicate the application of this solution to a number of problems of interest to structural engineers.

The method is first developed for the simplest problem of continuous beams, assuming constant moment of inertia, symmetrical loads, and freely supported ends. It is then shown how the method can be extended to problems of restrained ends, unsymmetrical loading, and varying moment of inertia, so as to yield a simple graphic solution for practically any problem involving restrained or continuous beams.

It is shown how the graphic solution can be applied to the determination of bending moments, shears, and reactions for any given loading, and to the construction of influence lines for moving loads.

The application of the method is further extended to the graphic solution of rigid portals and other indeterminate frames; to problems involving the settlement of supports; and to short-cut graphic determinations of secondary stresses in bridge trusses.

A number of concepts are introduced which can be applied to advantage in the solution of many other problems, and diagrams and tables are furnished to facilitate practical application.

Practical usefulness and clarity of presentation have been the governing considerations. The essentials of argument, method, and concept are presented in the body of the paper, and the necessary mathematical demonstrations are given in condensed form in an Appendix.

Introduction

The proper treatment of restrained and continuous beams is of constantly increasing importance in structural design. Before the general use of reinforced concrete, the application of the Theorem of Three Moments was required

Note.—Written discussion on this paper will be closed with the **January, 1926**, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

* A paper of this title was first submitted by Mr. Nishkian and reviewed for the Society by Mr. Steinman. Mr. Steinman, who had done independent work along parallel lines, suggested that a paper of larger scope and usefulness be undertaken jointly. By mutual agreement this was done resulting in this paper.

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only in special structures. To-day, concrete buildings and bridges cannot be economically designed without a careful analysis of the moments in the continuous beams and slabs. The present methods of such analysis, however, are so involved and tedious that most building ordinances have established arbitrary coefficients for moments in a continuous beam. Needless waste of material naturally results.

The writers are hopeful that an easy method for obtaining moments in continuous beams will gradually eliminate the use of arbitrary coefficients, thus giving the engineer an opportunity to exercise his ingenuity and judgment toward obtaining maximum efficiency and economy. The use of prescribed coefficients makes the work mechanical and unscientific. Arbitrary coefficients for continuous beams as given in most building ordinances result in moments which may be 50% too small or 60% too large.

The availability of simple, graphic solutions should also promote the increased use of certain efficient forms of reinforced concrete construction which come under the classification of "Indeterminate Frames." These structural forms have recently come into extensive use abroad in response to economic conditions which demanded maximum efficiency; but the development of the same class of structures in the United States appears to have been retarded by a prevailing impression that the required analysis is involved and tedious. The deterring influence of such an impression is generally under-estimated.

The methods presented in this paper should be helpful to the designer of steel structures as well as to the specialist in reinforced concrete. Of particular interest to the bridge engineer should be the proposal of methods for the expeditious determination of secondary stresses. The tediousness of the standard analytical methods has unquestionably been an obstacle to the more frequent investigation of these stresses in bridges. This paper offers some simple graphic methods that will quickly give the secondary stresses in any members of a truss, with sufficient accuracy for practical requirements. The availability of such methods removes the prevailing excuse for ignoring these important elements of structural stress, and should be conducive to improvements in specifications and in design.

Definitions and Symbols

The following notation and nomenclature are used throughout this paper, and are illustrated in Fig. 1.

R_1, R_2, R_3, ..., = supports in a continuous beam. The verticals through these points of support are called Reaction Lines, or R-lines.

l_1, l_2, l_3, ..., = span to right of R_1, R_2, R_3, ..., respectively. The vertical center lines of these respective spans are called C-lines. The one-third lines, or verticals through the one-third points of the respective spans, are called U and V-lines (U-line at left and V-line at right, respectively, in each span). By interchanging the one-third segments between the U and V-lines adjacent to each intermediate support, a new vertical is obtained near the R-line; this new vertical is called the Transposition Line, or

T-line. (For unequal moments of inertia, see Fig. 15; for varying moments of inertia, see Fig. 22.)

A_1, A_2, A_3, ..., = simple-beam moment areas on the span, l_1, l_2, l_3, ..., respectively; that is A_1 is the area of the moment graph for the applied loading on the span, l_1, if l_1 were a simple beam, etc. Verticals through the centers of gravity of these respective moment areas are called G-lines. For symmetrical loading in any span, the G-line coincides with the C-line, as indicated in Fig. 1.

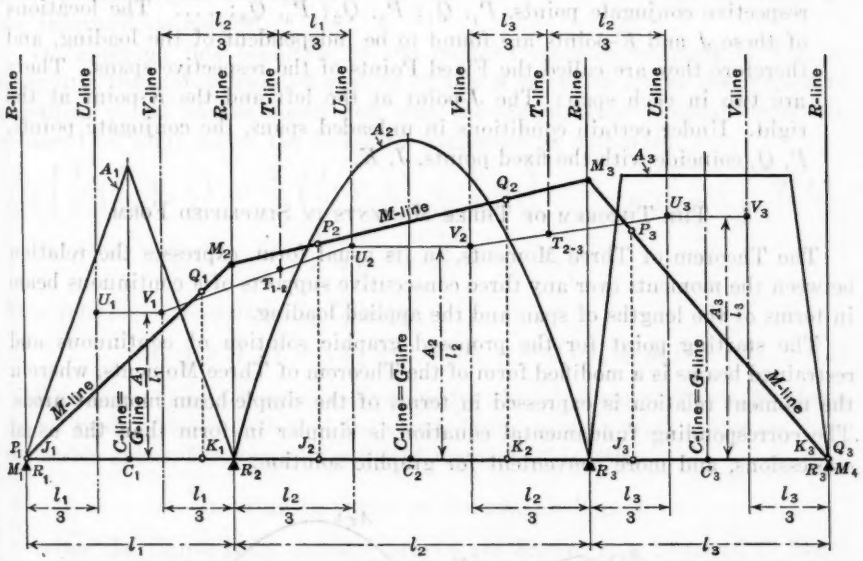


FIG. 1.—NOTATION REFERENCE DIAGRAM FOR THE METHOD OF CONJUGATE POINTS.

M_1, M_2, M_3, ..., = moments at supports, R_1, R_2, R_3, ..., respectively. If these moments are laid off (with reversed sign) on the respective R-lines and connected by a straight line in each span, the resulting line is the moment datum line and is called the Closing Line or M-line. The ordinates of the simple-beam moment graphs (A_1, A_2, A_3, ...), measured from this datum line, are the resultant bending moments sought.

U_1, V_1; U_2, V_2; U_3, V_3; ...; = points on the one-third verticals, or U and V-lines in the respective spans. The heights of these points, for symmetrical loading, equal $\frac{A_1}{l_1}$, $\frac{A_2}{l_2}$, $\frac{A_3}{l_3}$, ..., respectively. (For unsymmetrical loading, see Fig. 14; for varying moments of inertia, see Fig. 22.)

$T_{1\text{-}2}$, $T_{2\text{-}3}$, ..., = points on the T-lines where these are intersected by the straight lines joining the adjacent U-V points (the lines, V_1-U_2, V_2-U_3, ..., respectively).

P_1, Q_1; P_2, Q_2; P_3, Q_3; ...; = points, two in each span, on the moment datum line or closing line of the continuous beam. These are called the Conjugate Points. The P-points are on the left and the Q-points are on the right in the respective spans. Consecutive P-points are mutually

conjugate, each being obtainable from the preceding one by a simple graphic construction; similarly, consecutive Q-points are mutually conjugate. Commencing at the left end of the continuous beam, the P-points are successively located; commencing at the right end, the Q-points are successively located. Joining the P and Q-points in each span by a straight line extending to common intersections over the supports, the desired closing line (M-line) is obtained.

J_1, K_1; J_2, K_2; J_3, K_3; ...; = the projections on the base line, of the respective conjugate points, P_1, Q_1; P_2, Q_2; P_3, Q_3; .... The locations of these J and K-points are found to be independent of the loading, and therefore they are called the Fixed Points of the respective spans. There are two in each span: The J-point at the left and the K-point at the right. Under certain conditions in unloaded spans, the conjugate points, P, Q, coincide with the fixed points, J, K.

1.—The Theorem of Three Moments in Simplified Form

The Theorem of Three Moments, in its usual form, expresses the relation between the moments over any three consecutive supports of a continuous beam in terms of the lengths of span and the applied loading.

The starting point for the proposed graphic solution of continuous and restrained beams is a modified form of the Theorem of Three Moments, wherein the moment relation is expressed in terms of the simple-beam moment areas. The corresponding fundamental equation is simpler in form than the usual expressions, and more convenient for graphic solution.

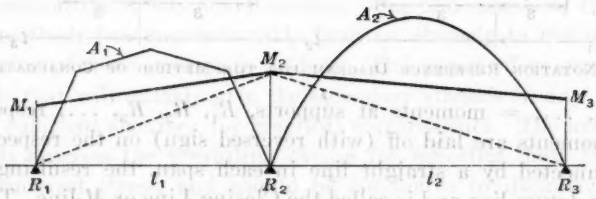


Fig. 2.—Moment Areas in Two Spans of a Continuous Beam.

Consider two consecutive spans of a continuous beam, as represented in Fig. 2. The simple-beam moment graphs, A_1 and A_2, are drawn above the base line in the respective spans. The moments at the supports, M_1, M_2, and M_3, due to continuity, are laid off on the respective reaction lines, and are then connected by straight lines in each span. These straight lines form a supplementary moment graph which must be added, algebraically, to the simple-beam moment graphs to give the resultant moments in the continuous spans. The total moment diagram in each span then consists of the sum of the M-trapezoid (which may be divided into two triangles as indicated) and the simple-beam moment graph, A_1 or A_2.

For the present, it is assumed that the loading is symmetrical in each span, and that the moment of inertia, I, of the beam is constant. (The modifications for unsymmetrical loads and varying I will be introduced subsequently.)

A simple way of writing the Equation of Three Moments is to apply one of the established principles of the well-known Method of Moment Areas, namely: The angular deflections at the ends of any span are equal to the respective end reactions produced by loading that span with its moment area, divided by $E\,I$. At the common support (R_2, Fig. 2), the two spans must have equal but opposite angular deflections, because the beam is continuous. Consequently, the total moment areas in the two spans (considered as imaginary loadings) must produce equal but opposite reactions (or a zero combined reaction) at the intermediate support.

On applying this condition to the moment areas represented in Fig. 2, observing that the centers of gravity of the M-triangles are at the respective one-third points of the spans and that the simple-beam moment areas are centered on their spans, the following expression is obtained:

$$\frac{1}{E\,I}\left[\frac{1}{3}\left(\frac{1}{2}\,M_1\,l_1\right)+\frac{2}{3}\left(\frac{1}{2}\,M_2\,l_1\right)+\frac{1}{2}\,A_1\right]=-\frac{1}{E\,I}\left[\frac{1}{3}\left(\frac{1}{2}\,M_3\,l_2\right)+\frac{2}{3}\left(\frac{1}{2}\,M_2\,l_2\right)+\frac{1}{2}\,A_2\right]$$

This relation is re-written as follows:

$$(M_1+2\,M_2)\,l_1+(M_3+2\,M_2)\,l_2=-3\,(A_1+A_2)\ldots\ldots\ldots\ldots(1)$$

Equation (1) is a general expression for the Theorem of Three Moments for all types of loads such that the simple-beam moment areas center on their respective spans, for beams of constant moment of inertia.

2.—The Concept of Conjugate Points

After the simple-beam moment graphs are drawn, it is required to locate the correct closing line, or M-line. The latter will serve as a datum line from which the resultant moment ordinates may be measured.

Evidently, the correct closing line must satisfy Equation (1). There are, however, an infinite number of lines that will satisfy this equation. Even if a point, P_1, is known as a point through which the correct closing line will pass, an infinite number of lines satisfying the equation can be drawn through P_1. Notwithstanding this difficulty, the following analysis leads to a solution of the problem.

In Fig. 3, let the two M-lines indicated represent any two lines passing through any given point, P_1, and satisfying Equation (1). Let P_2 be the second point of intersection of these two lines. From the geometry of Fig. 3 and the condition imposed by Equation (1), it is easy to establish the necessary relations between the co-ordinates, x_1, y_1, and x_2, y_2, of the two points. These relations are found to be expressed by Equations (2) and (3):

$$\left(3-\frac{l_1}{x_1}\right)l_1=\left(\frac{l_2}{x_2}-3\right)l_2\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{l_1^2\,y_1}{x_1}+\frac{l_2^2\,y_2}{x_2}=-3\,(A_1+A_2)\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(For the derivation of these equations, see the Appendix.)

Equation (2) fixes the abscissa relationship of the two points, P_1 and P_2. It should be noted that the abscissa relationship is independent of the applied loading and of the ordinates of the points. Equation (3) determines the ordinate relationship and expresses it in terms of the moment areas, A_1 and A_2.

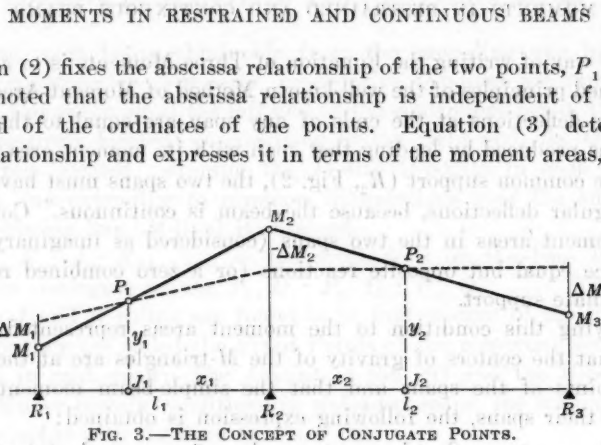


FIG. 3.—THE CONCEPT OF CONJUGATE POINTS.

If one of the points, P_1, is known, Equations (2) and (3) suffice to determine the location of the other point of intersection, P_2. As the two lines assumed were any two lines passing through P_1 and satisfying Equation (1), it follows that all lines passing through P_1 and satisfying Equation (1) will also pass through the point, P_2. Therefore, if the correct closing line passes through P_1, it will also pass through P_2. Accordingly, Equations (2) and (3) provide a means of determining successive points (in consecutive spans) through which the correct closing line will pass, after one such point is known.

Inspection of these governing equations (Equations (2) and (3)), shows that they are symmetrical and that the two points, P_1 and P_2, are interchangeable in each equation; hence the relations between the two points are reversible or mutual. As P_1 and P_2 are coupled by a definite, reciprocal relationship, they are appropriately named conjugate points.

3.—GRAPHIC CONSTRUCTION OF CONJUGATE POINTS

In order to arrive at a graphic solution of the problem, it is necessary to translate the governing equations (Equations (2) and (3)) into an equivalent geometrical construction.

The writers have devised a number of graphic constructions for the location of the conjugate points, and present the following as probably the simplest and most flexible solution.

To represent the influence of the applied loading, two points, U and V, are first marked in each span (see Fig. 1). These points are located on the one-third lines, at a height equal to $\frac{A}{l}$, that is,

$$U_1 = V_1 = \frac{A_1}{l_1};\ U_2 = V_2 = \frac{A_2}{l_2};\ \text{etc} \qquad (4)$$

By interchanging the one-third span segments between the U and V lines adjacent to each intermediate support, a new vertical is obtained near the reaction line; this new vertical is called the transposition line, or T-line (Fig. 1). (If two consecutive spans are equal, their T-line obviously coincides with the R-line.)

Where the T-line is intersected by the straight line joining the adjacent U-V-points, a new point is obtained which is called the T-point. Thus (Fig. 1), the line, V_1-U_2, locates the point, T_{1-2}, the line, V_2-U_3, locates the point, T_{2-3}, etc. The T-point is a significant point and represents, graphically, the combined influence of the loadings applied on the two adjoining spans. From the geometry of the construction, as indicated in Fig. 4, it is easily shown that the height of the T-point represents the important function:

$$T_{1-2} = \frac{A_1 + A_2}{l_1 + l_2} \qquad (5)$$

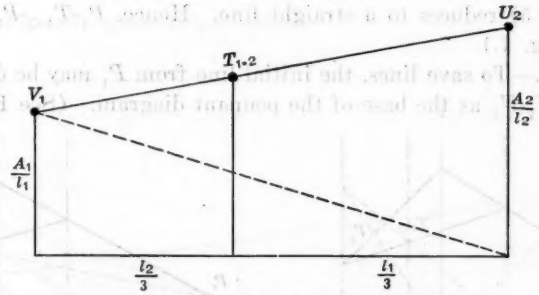


FIG. 4.—LOCATION OF THE T-POINT.

After the T-points are thus located, the construction proceeds, as follows: Let P_1 (Fig. 5) be a known point; it is required to find its conjugate point, P_2. Through P_1 and T_{1-2}, draw a "pennant diagram", $P_1\ H_1\ B\ H_2$ (Fig. 5), as follows: From P_1 draw a line in any direction, intersecting the $\frac{1}{3}$-line (V-line) at some point, H_1, and the reaction-line (R-line) at some point, B; from H_1 draw a line through T_{1-2}, intersecting the $\frac{1}{3}$-line (U-line) of the next span in some point, H_2; on the line joining B with H_2, the required point, P_2, will lie. It should be noted that the pennant diagram has its apices on the reaction line and the one-third lines, and its base passes through the T-point.

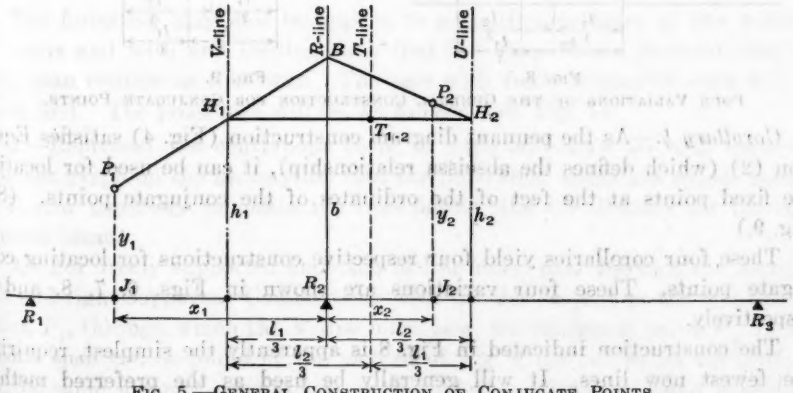


FIG. 5.—GENERAL CONSTRUCTION OF CONJUGATE POINTS.

The foregoing is a general graphic construction representing the relations expressed by Equations (2) and (3). (For proof, see the Appendix.)

From the proposition that this construction satisfies Equations (2) and (3), the following useful corollaries are at once deduced.

Corollary 1.—As the initial line, $P_1 H_1$ (Fig. 5), of the pennant diagram, may be drawn in any direction, a second pennant diagram starting from P_1 will intersect the first pennant diagram in the required point, P_2. (See Fig. 6.)

Corollary 2.—If the initial line from P_1 is taken through T_{1-2}, the pennant diagram (Fig. 5) reduces to a straight line. Hence, P_1-T_{1-2}-P_2 is a straight line. (See Fig. 7.)

Corollary 3.—To save lines, the initial line from P_1 may be drawn through V_1, utilizing V_1-U_2 as the base of the pennant diagram. (See Fig. 8.)

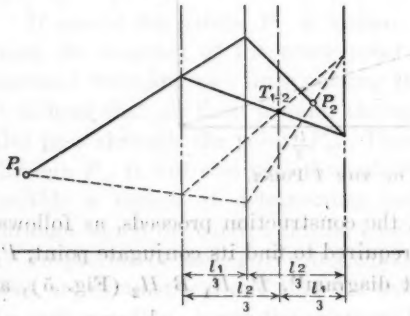


FIG. 6.

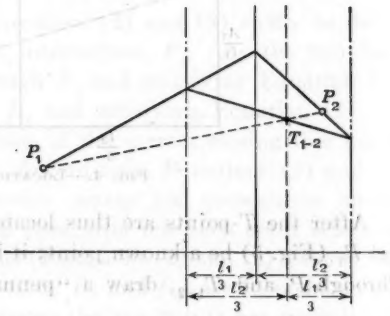


FIG. 7.

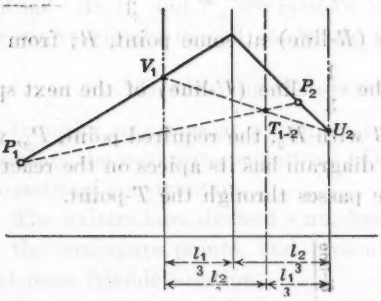


FIG. 8.

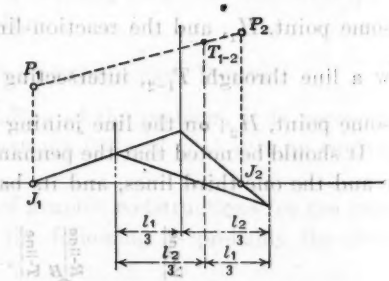


FIG. 9.

FOUR VARIATIONS OF THE GENERAL CONSTRUCTION FOR CONJUGATE POINTS.

Corollary 4.—As the pennant diagram construction (Fig. 4) satisfies Equation (2) (which defines the abscissa relationship), it can be used for locating the fixed points at the feet of the ordinates of the conjugate points. (See Fig. 9.)

These four corollaries yield four respective constructions for locating conjugate points. These four variations are shown in Figs. 6, 7, 8, and 9, respectively.

The construction indicated in Fig. 8 is apparently the simplest, requiring the fewest new lines. It will generally be used as the preferred method throughout the remainder of this paper.

The construction indicated in Fig. 9 offers advantages when more than one loading condition has to be considered, as in the construction of influence lines.

Some alternative constructions, which may be of interest to the reader, are indicated in Fig. 10. Instead of the $\frac{1}{3}$-lines and their transposition point, T_{1-2}, the $\frac{1}{2}$-lines (or C-lines) are used with a corresponding transposition point, S_{1-2}. The constructions of Figs. 6, 7, 8, and 9, however, appear to be simpler and of greater general usefulness.

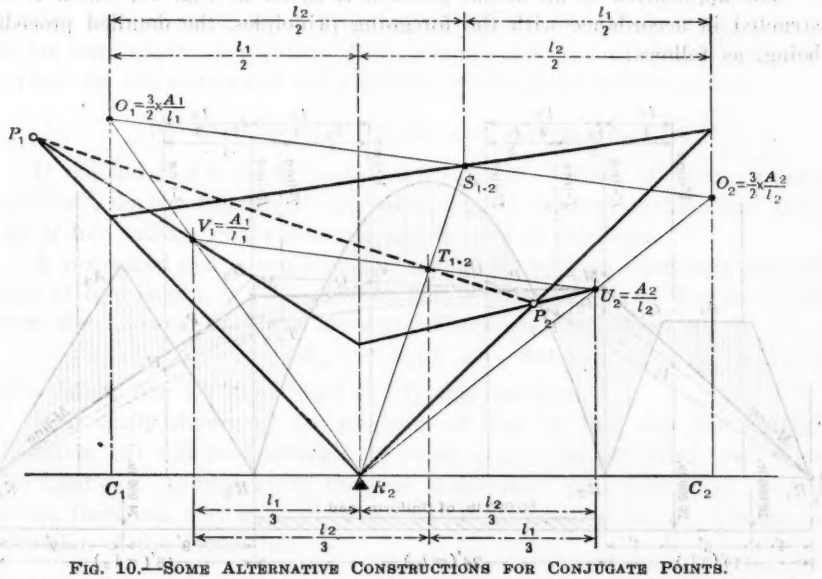


FIG. 10.—SOME ALTERNATIVE CONSTRUCTIONS FOR CONJUGATE POINTS.

4.—APPLICATION OF THE METHOD OF CONJUGATE POINTS

The foregoing may now be applied to a continuous beam of any number of spans and with any loading such that the simple-beam moment area on each span centers on that span. The case with freely supported ends will be taken first. The procedure will be, as follows (see Fig. 1):

1.—Represent the continuous beam to scale and draw on each span the moment graph on the assumption that the individual spans are simple beams. It is now necessary to locate the true datum line (or M-line) for the continuous beam.

2.—For freely supported ends the M-line must pass through the extreme left and right supports. Commencing with the left support as the first known point, P_1, through which the M-line must pass, the conjugate point, P_2, in the second span can be found by the method of Fig. 8 (or one of its variations). In the same manner, a point, P_3, in the third span can be found conjugate

to the point, P_2, in the second span. This will be repeated until the end span is reached.

3.—Beginning at the extreme right support and working to the left, another set of conjugate points, ..., Q_3, Q_2, Q_1, can be located in similar manner.

4.—There are now two points, P and Q, in each span, through which the M-line or datum line must pass. If each pair of points, P_1 and Q_1, P_2 and Q_2, P_3 and Q_3, ..., are connected by a straight line, and this line is extended in each span to the reaction verticals, the broken line thus formed is the required M-line, or true datum line. Common intercepts on the intermediate support verticals will check the correctness of the work.

The application to an actual problem is given in Fig. 11, which is constructed in accordance with the foregoing principles, the detailed procedure being, as follows:

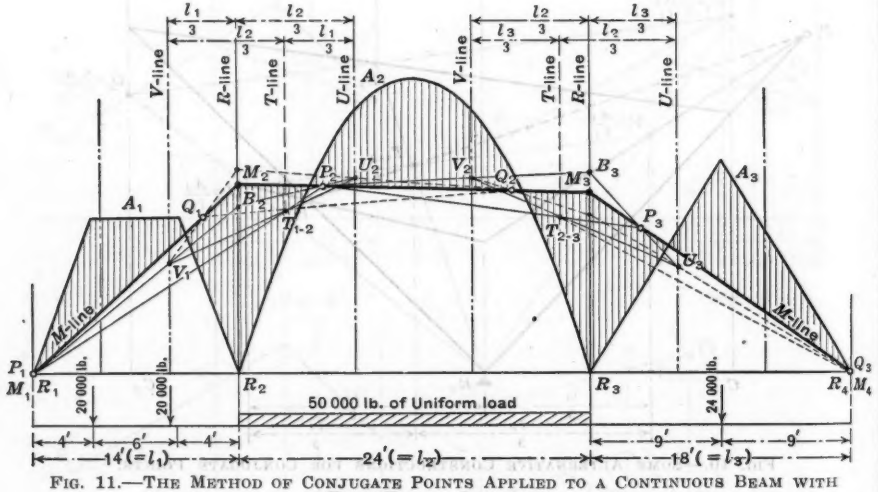


FIG. 11.—THE METHOD OF CONJUGATE POINTS APPLIED TO A CONTINUOUS BEAM WITH ENDS FREELY SUPPORTED.

(*a*) Draw the ordinary moment graphs, A_1, A_2, A_3, in the respective spans, assuming the latter to be simple beams.

(*b*) On the $\frac{1}{3}$-lines of each span locate the U-V points, such that:

$$U_1 = V_1 = \frac{A_1}{l_1}; \quad U_2 = V_2 = \frac{A_2}{l_2}; \quad U_3 = V_3 = \frac{A_3}{l_3}.$$

(*c*) Locate the T-line near each intermediate reaction vertical by transposing the adjoining one-third span segments.

(*d*) Connect V_1-U_2 and V_2-U_3. Where these connecting lines cut the T-lines, mark the respective T-points, $T_{1\text{-}2}$ and $T_{2\text{-}3}$.

(*e*) Draw P_1-V_1 intersecting the R_2-vertical at B_2. Draw B_2-U_2. A line from P_1 through $T_{1\text{-}2}$ intersects B_2-U_2 in the conjugate point, P_2.

(*f*) Draw P_2-V_2 intersecting the R_3-vertical at B_3. Draw B_3-U_3. A line from P_2 through $T_{2\text{-}3}$ intersects B_3-U_3 in the conjugate point, P_3.

(*g*) The conjugate points, Q_2 and Q_1, may be located in similar manner by starting at the other end, Q_3, and working toward the left.

(*h*) Connect P_1-Q_1, P_2-Q_2, and P_3-Q_3. If these lines have common intercepts on the R and R-verticals, the correctness of the work is checked.

(*i*) The broken line last drawn is the required closing line, or M-line. Vertical intercepts between this datum line and the previously drawn simple-beam moment graphs represent the bending moments at the respective points of the beam. These intercepts are indicated by the areas shaded in Fig. 11. Ordinates above the closing line represent positive bending moments, and ordinates below the closing line represent negative bending moments.

It should be noted that the closing line represents the moments of continuity with reversed sign. This reversal of the M-line about the base axis is for convenience, permitting direct graphic subtraction of the moments to replace the less convenient addition that would otherwise be required.

5.—Application to Beams with Restrained Ends

If the end of a beam is restrained, or "fixed", instead of freely supported, or "free", the construction illustrated in Fig. 11 requires modification because the M-line will no longer pass through the end of the beam.

A restrained end is equivalent to continuity with an imaginary adjoining span of zero length, $l_0 = 0$, and zero end moment, $M_0 = 0$. For such condition, the Theorem of Three Moments, Equation (1), reduces to:

$$(M_2 + 2\,M_1)\,l_1 = -\,3\,A_1 \qquad (6)$$

The datum line (M-line) must satisfy this equation.

It is easily shown, by the geometry of Fig. 12, that any line satisfying Equation (6) will pass through the point, U_1, on the one-third vertical near the fixed end. Consequently, the true M-line must pass through U_1, and this point, therefore, may be used as the initial conjugate point, P_1, for the construction of the datum line.

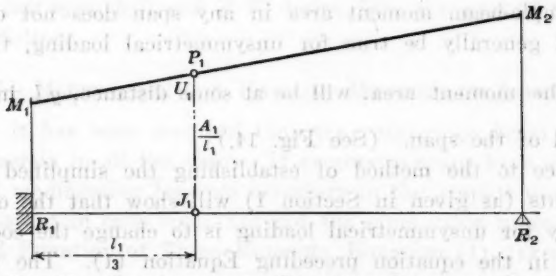


Fig. 12.—Construction for Beam with Restrained End.

It follows that for any continuous beam with fixed ends the same procedure may be followed as that given for free ends, except that the initial point must be taken at U_1 instead of R_1. Similarly, if the extreme right end of the beam is fixed, the last V-point must be used instead of the last R-point as an initial conjugate point.

Fig. 13 shows the solution of the problem illustrated in Fig. 11 on the assumption that the ends are fixed. The procedure is identical with that followed in the construction of Fig. 11, except that the initial conjugate points, P_1 and Q_3, are taken at the extreme U-V points (U_1 and V_3, respectively), instead of at the extreme R-points (R_1 and R_4, respectively). The resultant bending moments are represented by the vertical intercepts of the shaded areas in Fig. 13.

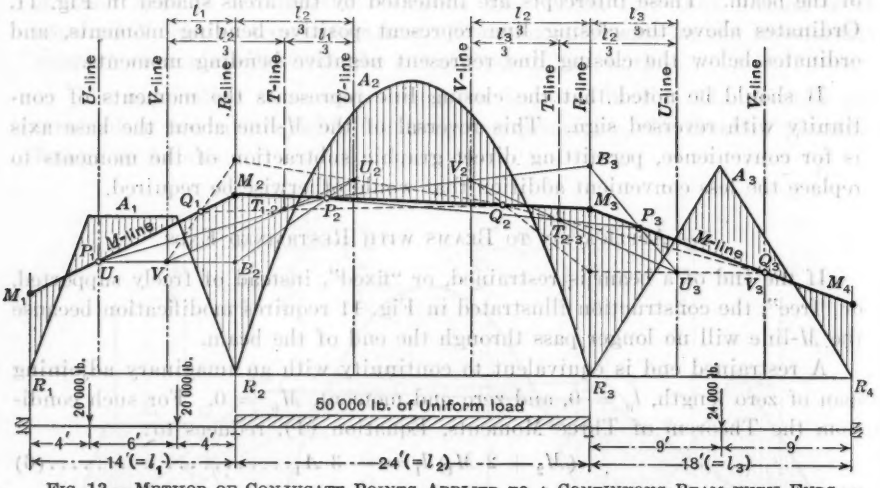


FIG. 13.—METHOD OF CONJUGATE POINTS APPLIED TO A CONTINUOUS BEAM WITH ENDS RESTRAINED.

6.—MODIFICATION FOR UNSYMMETRICAL LOADING

In the foregoing, it has been assumed that the simple-beam moment area in each span centered on that span. The modification necessary for unsymmetrical loads will now be given.

If the simple-beam moment area in any span does not center on the span, as will generally be true for unsymmetrical loading, the G-line, or centroid of the moment area, will be at some distance, $g\,l$, instead of $\frac{1}{2}\,l$, from the end of the span. (See Fig. 14.)

A reference to the method of establishing the simplified Equation of Three Moments (as given in Section 1) will show that the only modification necessary for unsymmetrical loading is to change the coefficient of A from $\frac{1}{2}$ to g in the equation preceding Equation (1). The three-moment relation then becomes:

$$(M_1 + 2\,M_2)\,l_1 + (M_3 + 2\,M_2)\,l_2 = -\,6\,(g_1\,A_1 + g_2\,A_2) \ldots\ldots\ldots (7)$$

Equation (7) is the simplified Theorem of Three Moments generalized to include the case of unsymmetrical loading.

Only slight modification of the graphic solution is necessary to satisfy this modification of the fundamental equation. The only change necessitated in

the construction is a revised height of the U-V points, and is shown in Fig. 14. Mark a point, O, at the height, $\frac{A}{l}$, on the center line of the span; two lines drawn through O from the ends of the span will intercept the G-line at the required heights of U and V. The remainder of the graphic solution is exactly as given heretofore.

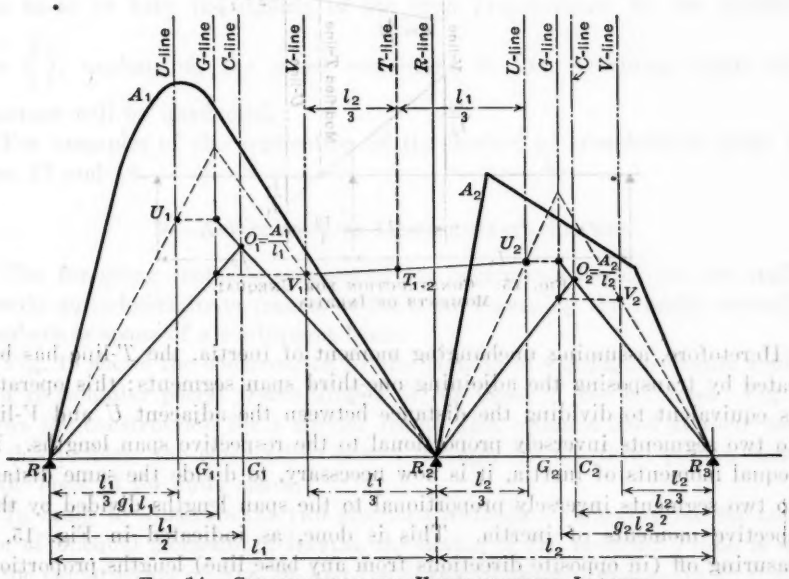


FIG. 14.—CONSTRUCTION FOR UNSYMMETRICAL LOADING.

The logic of this construction is almost obvious from a comparison of Equation (7) with Equation (1); but a detailed proof is given in the Appendix.

As a check on the correctness of the construction, it may be noted (Fig. 14) that the two rays, R-U and R-V, drawn from the ends of the span, must meet on the G-line.

7.—MODIFICATION FOR UNEQUAL MOMENTS OF INERTIA

Thus far it has been assumed that the continuous beam has the same moment of inertia in all the spans. If successive spans have differing values of I, a slight modification becomes necessary in the graphic construction.

For I constant in each span, but not the same value in successive spans, the simplified Equation of Three Moments, Equation (1), takes the form:

$$(M_1 + 2M_2)\frac{l_1}{I_1} + (M_3 + 2M_2)\frac{l_2}{I_2} = -3\left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \quad \ldots\ldots\ldots (8)$$

in which, I_1 and I_2 are the moments of inertia in the respective spans. It may be noted that Equation (8) differs from Equation (1) only in the retention of the appropriate value of I as a denominator in each term; if $I_1 = I_2$, Equation (8) obviously reduces to the simpler form of Equation (1).

This amplification of the fundamental equation is satisfied by a slight modification in the graphic solution. The only change necessary in the construction is an altered location of the T-line; the location of the "modified T-line" is shown in Fig. 15.

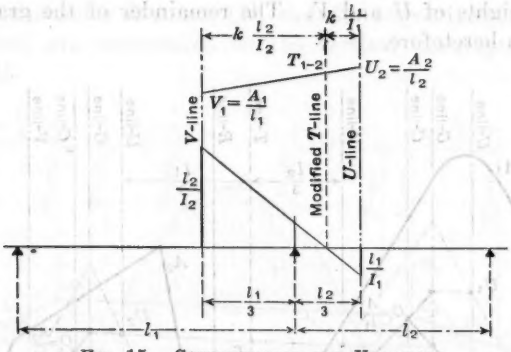


FIG. 15.—CONSTRUCTION FOR UNEQUAL MOMENTS OF INERTIA.

Heretofore, assuming unchanging moment of inertia, the T-line has been located by transposing the adjoining one-third span segments; this operation was equivalent to dividing the distance between the adjacent U and V-lines into two segments inversely proportional to the respective span lengths. For unequal moments of inertia, it is now necessary, to divide the same distance into two segments inversely proportional to the span lengths divided by their respective moments of inertia. This is done, as indicated in Fig. 15, by measuring off (in opposite directions from any base line) lengths proportional to $\frac{l_2}{I_2}$ and $\frac{l_1}{I_1}$ on the V and U-lines, respectively; the intersection of the connecting line with the base line locates the required modified T-line.

Except for this simple modification in the location of the T-lines, the graphic construction heretofore presented remains unchanged for unequal moments of inertia. (For proof, see the Appendix.)

It is readily seen that this construction for locating the T-line (Fig. 15) is more general and reduces to the previous construction (Fig. 4) as a special case when $I_1 = I_2$.

8.—THE METHOD OF "TRANSFORMED SPANS"

As an alternative to the modified T-line method of allowing for unequal values of I, represented in Fig. 15, a procedure which will be referred to as the Method of Transformed Spans, may sometimes be preferred. This procedure consists simply in altering the length scale of some of the spans in the drawing.

The idea is to change the delineated lengths of the spans in such manner as to produce a virtual condition equivalent to equal moments of inertia. Taking the value of I of the principal loaded span as a standard, any span having a lower value of I is lengthened, and any span having a higher value

of I is shortened (in Fig. 15), the lengthening or shortening being in the ratio of the standard value of I to the I of the span being altered. When this change is made in the delineated span lengths, the graphic solution can proceed the same as for equal moments of inertia.

The correctness of this method is founded on the following principle: If the length, l, and moment of inertia, I, of any span are altered in the same ratio so as to keep the rigidity of the span $\left(\text{represented by the quotient, } r = \frac{I}{l}\right)$, unchanged, the stress conditions in the remaining spans of a structure will be unaffected.

For examples of the application of the method of transformed spans, see Figs. 17 and 18.

9.—Application to Indeterminate Frames

The foregoing methods established for continuous beams can be applied directly to indeterminate frames or bents by regarding the rigidly connected members as spans of a continuous beam.

Fig. 16 shows the construction by the Method of Conjugate Points applied to a rectangular portal frame hinged at the bases of the columns. Fig. 17 shows the construction for a similar frame with columns fixed at their bases. In this problem the closing line passes through the one-third points, instead of the ends, of the outer spans.

In order to illustrate the two alternate methods of allowing for the condition of unequal moments of inertia, the device of the modified T-line is used in the construction of Fig. 16, and the device of transformed spans in that of Fig. 17. If the frames were trapezoidal (sloping columns) instead of rectangular, the constructions given in Figs. 16 and 17 would remain unchanged.

In Figs. 16 and 17, the frames and the applied loading are assumed to be symmetrical; consequently, the graphic construction needs to be carried out for only one side of the structure, as indicated. If the structure or loading is unsymmetrical, a small secondary correction must be introduced on account of the slight lateral displacement of the middle span; this displacement is equivalent to a yielding of the respective supports of the outer spans. (See Section 17.)

The method of conjugate points can also be applied advantageously to more complex indeterminate frames in which more than two members meet at a joint. Fig. 18 is an illustration; it represents a three-span continuous beam with rigidly connected indeterminate columns.

The construction shown in Fig. 18 introduces a device which will be referred to as the Composition of Rigidities. The flexural rigidity of a member is proportional to $\frac{E\,I}{l}$; if E is uniform, the rigidity may be represented by the quotient, $r = \frac{I}{l}$, or by $r = \frac{3}{2}\,\frac{I}{l}$ when the outer end of the member is

restrained. If two or more members are framed into a single joint, they may be replaced, in effect, by a single member having a rigidity equal to the sum of the respective individual rigidities.

The foregoing principle is utilized in the construction of Fig. 18. The

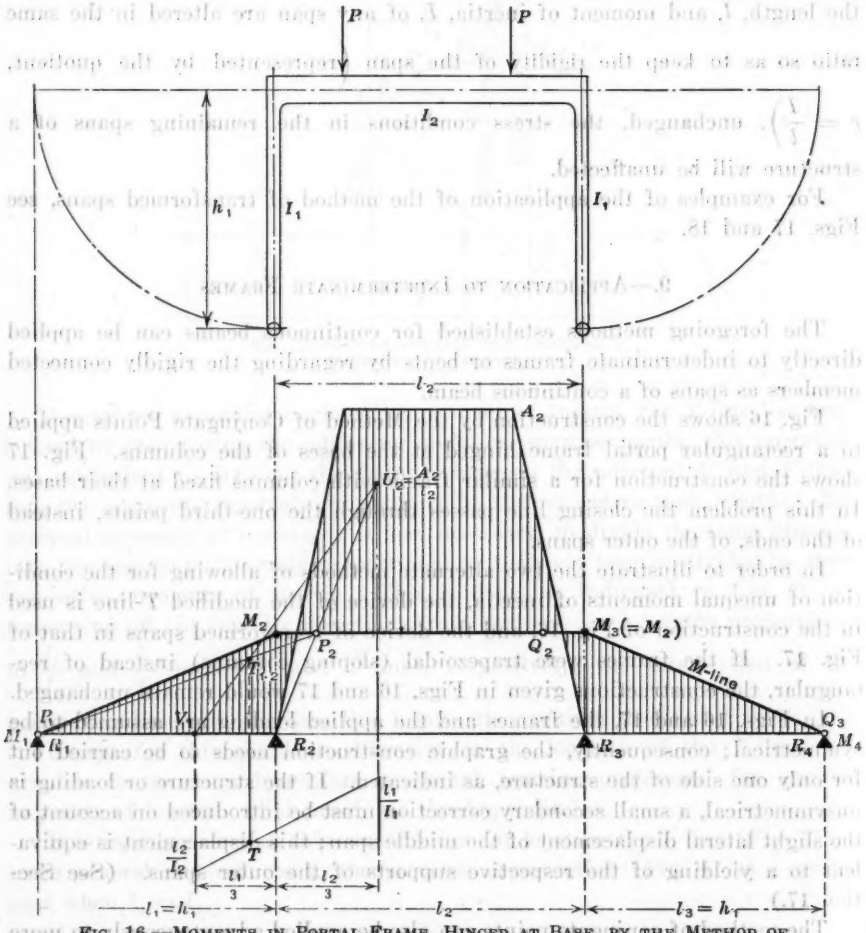


FIG. 16.—MOMENTS IN PORTAL FRAME, HINGED AT BASE, BY THE METHOD OF "MODIFIED T-LINES".

two members, l and h, at each end of the main span, l_2, are replaced by an equivalent single member, l_1. The rigidity of this equivalent single member must be:

$$r_1 = r_l + r_h$$

The nominal length of this member must then be:

$$l_1 = \frac{r_2}{r_1} \cdot l_2$$

in order that its nominal moment of inertia may be equal to the moment of inertia of the main span. (This is an application of the device of transformed spans.) With the side-span lengths, l_1, thus determined, the construction in Fig. 18 is identical with the construction heretofore presented for continuous beams and simple frames. (Compare Fig. 18 with Figs. 16 and 17.)

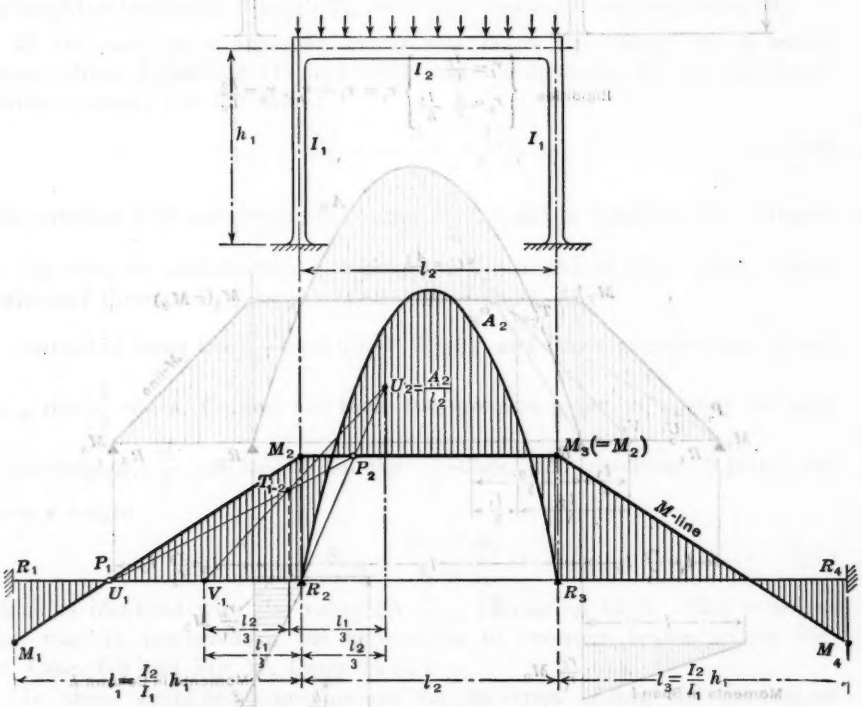


FIG. 17.—MOMENTS IN PORTAL FRAME, FIXED AT BASE, BY THE METHOD OF "TRANSFORMED SPANS".

On completion of the construction, the end moments found for the substituted equivalent members must be resolved or subdivided to give the bending moments in the respective component members. The total end moment, M_2 (Fig. 18), must be divided between the original component members in direct proportion to their respective rigidities. This final resolution of bending moments is indicated in Fig. 18; for the member with free end, the M-line goes to the free end; for the member with fixed end, the M-line goes through the outer one-third point.

This construction by the Composition of Rigidities has been explained at length because it has many useful applications; its utilization in the graphic determination of secondary stresses will be given in a subsequent section (Section 18).

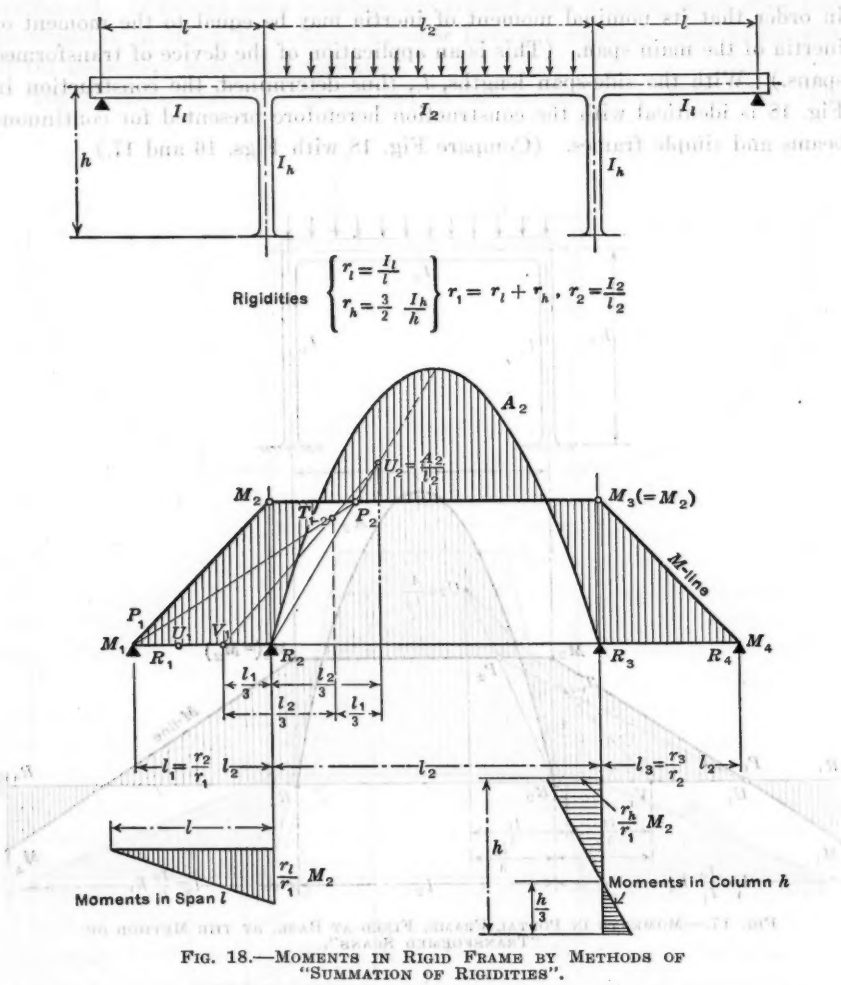


FIG. 18.—MOMENTS IN RIGID FRAME BY METHODS OF "SUMMATION OF RIGIDITIES".

10.—SIMPLIFICATION FOR TWO-SPAN BEAMS

For the common example of a two-span continuous beam, the foregoing constructions by the Method of Conjugate Points yield to a material simplification.

If the ends of a two-span continuous beam are free, the end moments have the values, $M_1 = 0$ and $M_3 = 0$. On substituting these values in Equation (1), that equation reduces to:

$$M_2 = -\frac{3}{2}\frac{A_1 + A_2}{l_1 + l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Comparing this with the value of T_{1-2} given by Equation (5), it is observed that the height of the peak of the closing line (the point, M_2) is $\frac{3}{2}$ the height

of the T-point. The problem is thereby solved without requiring the location of the conjugate points.

For this case, it is convenient to mark the U-V points at $\frac{3}{2}$ their usual height, $\frac{3}{2}\frac{A}{l}$, instead of $\frac{A}{l}$, so that the resulting T-point will be $\frac{3}{2}$ its customary height; a horizontal through T_{1-2} will then locate the required point, M_2.

If the ends of a two-span continuous beam are "fixed", it is easily shown (from Equations (1) and (6)) that the moment, M_2, at the intermediate support has the value:

$$M_2 = -\frac{A_1 + A_2}{l_1 + l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This equation does not contain the factor, $\frac{3}{2}$, found in Equation (9). Hence, in this case, no modification is required in the height of the T-point; and a horizontal through T_{1-2} locates the required point, M_2.

Instead of using the $\frac{1}{3}$-lines (U-V lines) and their transposition point, T_{1-2}, the $\frac{1}{2}$-lines (C-lines) and their transposition point, S_{1-2}, may be used. If the heights, $\frac{A}{l}$, are laid off on the $\frac{1}{2}$-lines, the resulting S-point will have a height:

$$S_{1-2} = \frac{A_1 + A_2}{l_1 + l_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

which is identical with the value for T_{1-2} (Equation (5)). This substitution may be preferred in the application to two-span beams. (See Fig. 19, Cases 6-9 and Fig. 20, Cases 14-17.)

In these simplified constructions for two-span beams, the corrections for unsymmetrical loads and for unequal moments of inertia are made in the same manner as heretofore established for the general case, namely, as indicated in Figs. 14 and 15.

The construction is still further simplified when the number of spans is reduced to one. For a single-span beam with one end restrained, Equation (9) reduces to $M = -\frac{3}{2}\frac{A_1}{l_1}$. (See Fig. 19, Case 1.) For a single-span beam with both ends restrained, Equation (10) reduces to $M = -\frac{A_1}{l_1}$. (See Fig. 19, Case 3.) These two results also follow directly from the previously established general relation for restrained ends indicated in Fig. 12.

11.—Application to Special Cases Commonly Occurring in Practice

Applications of the foregoing constructions to the special cases that occur most frequently in practice are indicated in a series of diagrams on Figs.

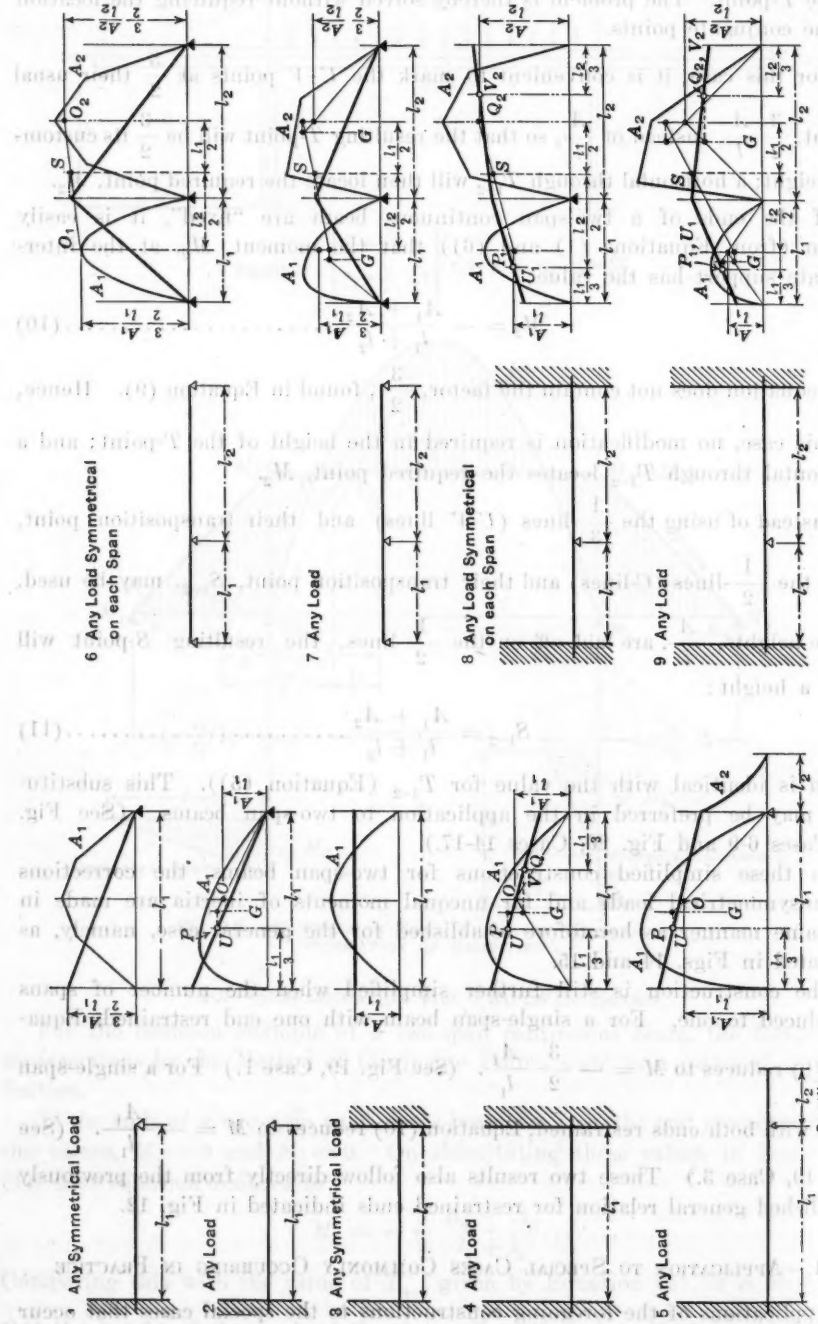


FIG. 19.—APPLICATION OF THE METHOD OF CONJUGATE POINTS TO SPECIAL CASES COMMONLY OCCURRING IN PRACTICE.

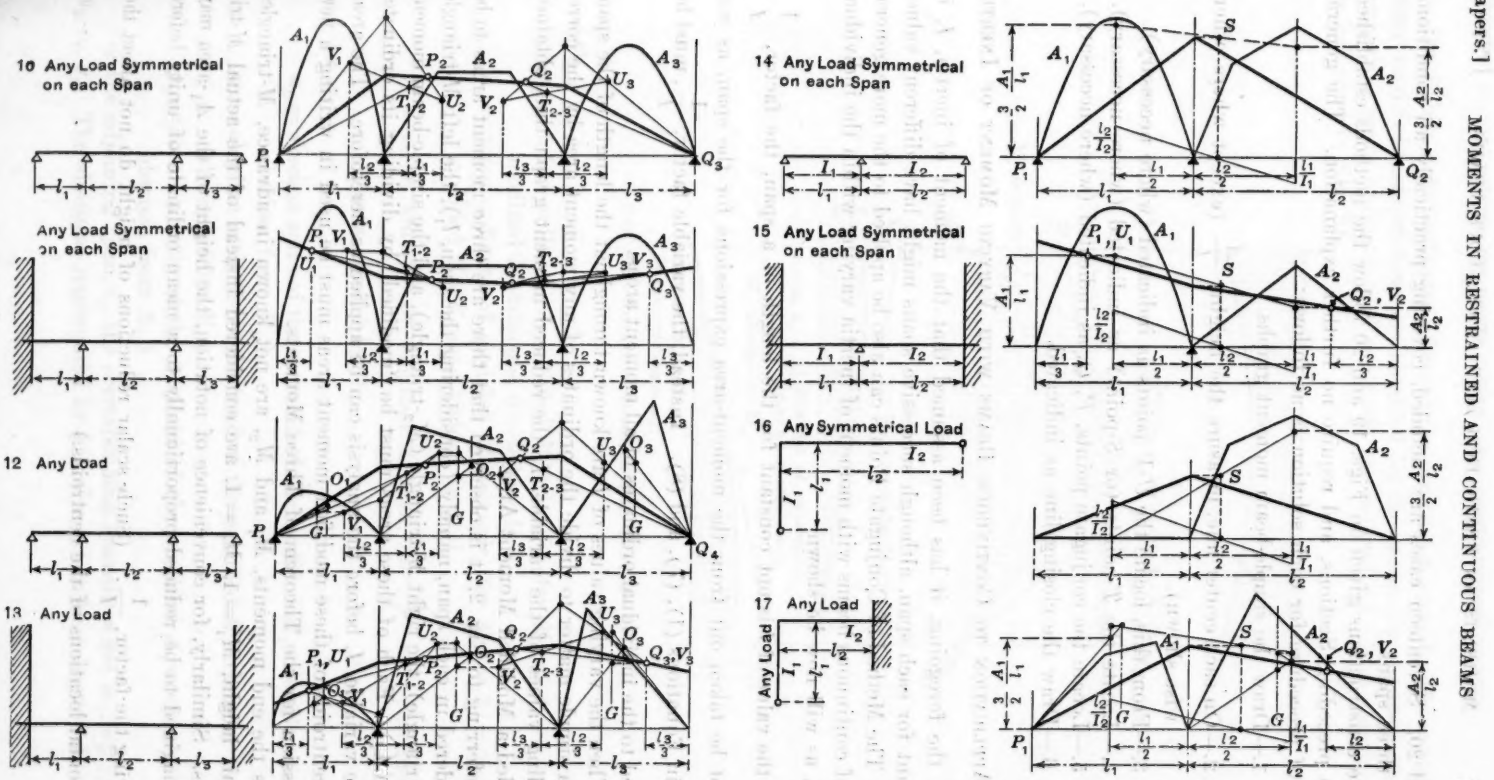

10 Any Load Symmetrical on each Span
11 Any Load Symmetrical on each Span
12 Any Load
13 Any Load
14 Any Load Symmetrical on each Span
15 Any Load Symmetrical on each Span
16 Any Symmetrical Load
17 Any Load
Any Load

FIG. 20.—APPLICATION OF METHOD OF CONJUGATE POINTS TO SPECIAL CASES COMMONLY OCCURRING IN PRACTICE.

19 and 20. Seventeen cases are included, covering practically all conditions up to three spans.

The constructions given on Figs. 19 and 20 follow the methods established in the preceding Sections, and require no further explanation. The general order of procedure for each solution is as follows:

1.—Draw the simple-beam moment graphs.

2.—On each center line, measure the height, $\frac{A}{l}$ (except where otherwise shown).

3.—From this, locate the U-V points as indicated (where necessary).

4.—Locate the T-points (or S-point) as indicated (where necessary).

5.—Locate the conjugate points, P, Q, as indicated (where necessary).

6.—Draw the closing line as indicated.

12.—Application to Continuous Beams with Varying Moment of Inertia

In the foregoing, it has been assumed that the moment of inertia, I, is constant for each span, although successive spans might have different values of I. The Method of Conjugate Points can also be applied to the more general case of continuous beams with moment of inertia varying within the individual spans, as will now be shown.

If the value of I is not constant for the length of a span, the factor, $\frac{1}{I}$, cannot be taken out from the moment-area expressions for the span, as was done in Equations (1), (7), and (8). Instead, the variable factor, $\frac{1}{I}$, must be applied to the individual ordinates of all moment areas.

When the varying values of I are known throughout the length of the span, it is a simple matter to divide the ordinates of any moment area by the corresponding values of the variable, I; the reduced moment graph thus obtained is called a Modified Moment Area.

Referring to Fig. 2, it is observed that there are three moment areas to be considered in each span, namely (considering the span, l_1), the left M-triangle (M_1-triangle), the right M-triangle (M_2-triangle), and the simple-beam moment area (A_1). Each of these areas must be "modified" by dividing its ordinates by the variable, I, before the analysis can be applied as heretofore. The areas and centroids of these modified moment areas must be used in writing a new expression for the Theorem of Three Moments.

As the end moments, M_1 and M_2, are not known in advance, M-triangles of unit height, $M_1 = 1$, $M_2 = 1$, are considered instead of the actual M-triangles. Similarly, for convenience of notation, the height of the A_1-area may be imagined to be reduced proportionally to a mean ordinate of unity before applying the factor, $\frac{1}{I}$. (Such scalar reductions of height do not affect the horizontal locations of the centroids.)

On dividing the ordinates of these three moment areas by the variable, I, three modified moment areas are obtained for the span. By summations, their respective mean ordinates and controids are determined. The notation used is indicated in Fig. 21.

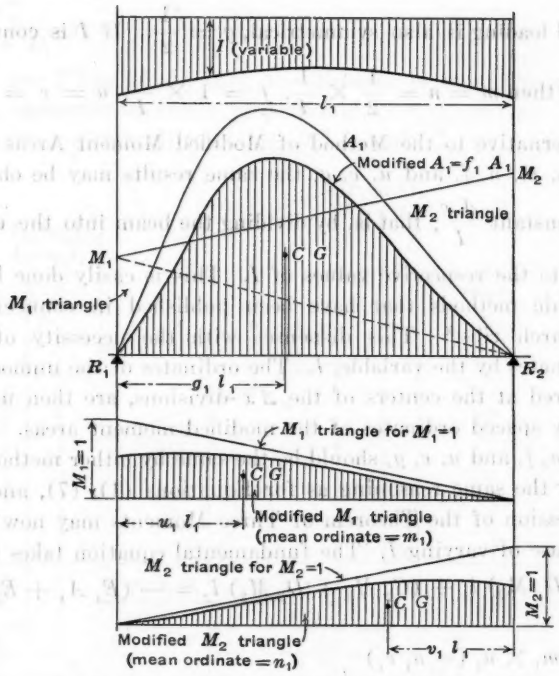


FIG. 21.—NOTATION DIAGRAM FOR VARYING I.

Let,

m_1 = mean ordinate of modified M_1-triangle, for $M_1 = 1$; = area of modified M_1-triangle, divided by $M_1 l_1$.

n_1 = mean ordinate of modified M_2-triangle, for $M_2 = 1$; = area of modified M_2-triangle, divided by $M_2 l_1$.

f_1 = mean ordinate of modified A_1-area, for $\frac{A_1}{l_1} = 1$; = modified A_1-area divided by A_1.

and, let,

u_1 = abscissa ratio of the centroid of the modified M_1-triangle. ($u_1 l_1$ = abscissa from R_1.)

v_1 = abscissa ratio of the centroid of the modified M_2-triangle. ($v_1 l_1$ = abscissa from R_2.)

g_1 = abscissa ratio of the centroid of the modified A_1-area. ($g_1 l_1$ = abscissa from R_1.)

For the adjoining span, l_2, the corresponding quantities are m_2, n_2, f_2, and u_2, v_2, g_2. The abscissas represented by u and v are to be measured from the

respective ends of the span, $u\,l$ from the left end and $v\,l$ from the right end; the abscissas represented by g are to be measured from the left end of each span. (If the span is symmetrical in variation of section, $m = n$ and $u = v$. If the applied loading is also symmetrical, $g = \frac{1}{2}$. If I is constant throughout the span, then $m = n = \frac{1}{2} \times \frac{1}{I}$, $f = 1 \times \frac{1}{I}$, $u = v = \frac{1}{3}$.)

As an alternative to the Method of Modified Moment Areas for obtaining the constants, m, n, f, and u, v, g, the same results may be obtained by the Method of Constant $\frac{\Delta x}{I}$, that is, by dividing the beam into the elements, Δx, proportional to the respective values of I. This is easily done by any one of several graphic methods that have been published in connection with the analysis of arch ribs.* This dispenses with the necessity of dividing all moment ordinates by the variable, I. The ordinates of the unmodified moment areas, measured at the centers of the Δx-divisions, are then used instead of the uniformly spaced ordinates of the modified moment areas. The resulting values of m, n, f, and u, v, g, should be the same by either method.

Following the same reasoning as for Equations (1), (7), and (9), a more general expression of the Theorem of Three Moments may now be written to include the case of varying I. The fundamental equation takes the form:

$$(C_1 M_1 + D_1 M_2)\, l_1 + (C_2 M_2 + D_2 M_3)\, l_2 = -(E_1 A_1 + E_2 A_2) \ldots . (12)$$

in which,

$$C_1 = m_1 \times u_1 \,(= n_1 v_1)$$
$$D_1 = n_1 \,(1 - v_1)$$
$$E_1 = f_1 \times g_1$$
$$C_2 = m_2 \,(1 - u_2)$$
$$D_2 = n_2 \times v_2 \,(= m_2 u_2)$$
$$E_2 = f_2 \,(1 - g_2)$$

In application to any actual problem, with the I-variation and loading known, the numerical values of m, n, f, and u, v, g, are first computed for each span by a simple tabulation and summation of the known variables; and the products, C, D, E, are then inserted in Equation (12) to obtain the governing equation for each pair of spans.

Equations (1), (7), and (8) are only special cases of the more general Equation (12), as will be found on substituting the appropriate special values of the coefficients, C, D, and E.

The Method of Conjugate Points can be applied to the solution of Equation (12) in exactly the same manner as heretofore given, if the U, V, and T-points are located with due regard to the changed coefficients in the equation. It is easily shown that these new coefficients are satisfied if the U, V, and T-points are located as follows (see Fig. 22):

* "Plain and Reinforced Concrete Arches", Melan-Steinman, pp. 45–46.

1.—Locate all U and V-lines at the centroids of the modified M-triangles; that is, locate each U-line at the distance, $u\ x\ l$, from the left end of the span and each V-line at the distance, $v\ x\ l$, from the right end of the span. (Note that $v_1 = \dfrac{C_1}{C_1 + D_1}$, $u_2 = \dfrac{D_2}{C_2 + D_2}$.)

2.—Locate the T-line by dividing the distance between the adjacent U-V lines into two segments inversely proportional to $n_1\ l_1$ and $m_2\ l_2$; or, equivalently, inversely proportional to $(C_1 + D_1)\ l_1$ and $(C_2 + D_2)\ l_2$.

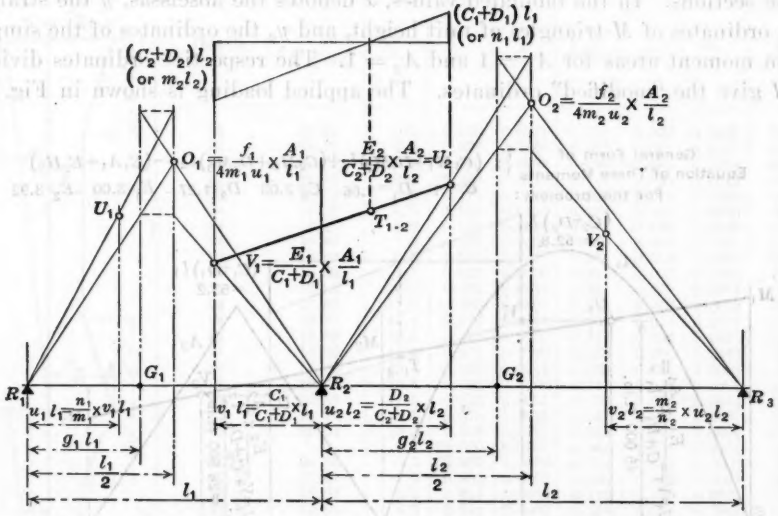


FIG. 22.—CONSTRUCTION FOR VARYING I, UNSYMMETRICAL SPANS AND LOADING.

3.—Locate the U-V points at the heights:

$$V_1 = \frac{E_1}{C_1 + D_1} \times \frac{A_1}{l_1} = \frac{f_1\, g_1}{n_1} \times \frac{A_1}{l_1}$$

$$U_2 = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2} = \frac{f_2\,(1 - g_2)}{m_2} \times \frac{A_2}{l_2}$$

The remainder of the procedure (for locating the closing line) is identical with that heretofore presented. (For proof, see the Appendix.)

If the beam and the loading in any span are symmetrical about the center line, the U and V-points in the span will be symmetrically located at equal heights. If unsymmetrical conditions obtain, Equation (12) is re-written for each successive pair of spans, and the new values of the numerical coefficients, C, D, and E, are used in locating the new group of V, U, and T-points; or the relations indicated in Fig. 22 may be used to locate the U and V-points.

If the special coefficients of Equations (1), (7), and (8) are substituted for C, D, and E in the foregoing rules, the latter will be found to reduce to the simpler rules previously established for the respective special cases.

A reduction of all the coefficients in Equation (12), by dividing through by any numerical factor, will not affect the construction, as the foregoing rules involve only ratios, and not absolute values, of the coefficients.

13.—Method for Varying I Applied to a Numerical Example

The foregoing method for varying I will now be illustrated by application to an actual numerical problem.

The profile of a two-span concrete beam is indicated in Table 1, which also gives the necessary tabulations and summations. The beam is assumed to be divided into sections 1 ft. long, and all variables refer to the mid-points of these sections. In the tabulated values, x denotes the abscissas, y the straight line ordinates of M-triangles of unit height, and y_a the ordinates of the simple-beam moment areas for $A_1 = 1$ and $A_2 = 1$. The respective ordinates divided by I give the "modified" ordinates. The applied loading is shown in Fig. 23.

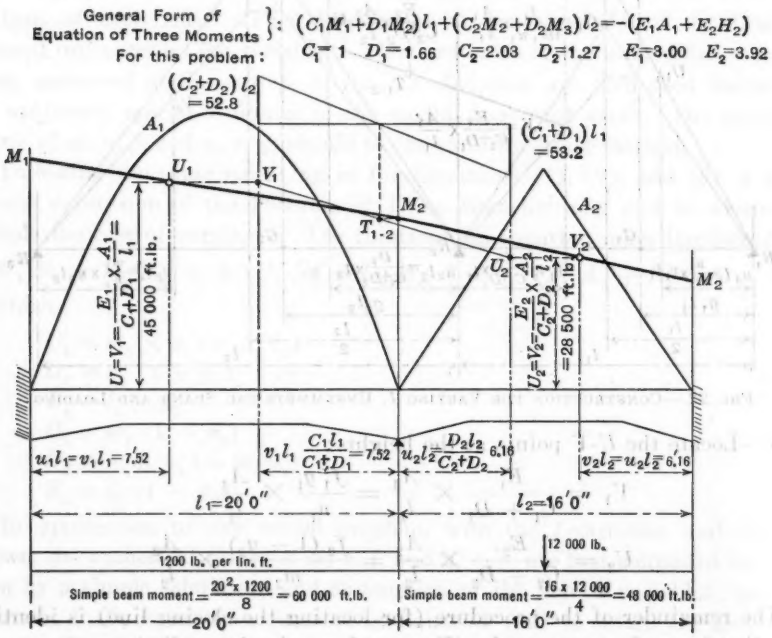


Fig. 23.—Method for Varying I, Applied to a Numerical Problem. (See Table 1.)

From Table 1:

$C_1 = 0.0289 \times 0.376 = 0.0109$
$D_1 = 0.0289 \times (1 - 0.376) = 0.0181$
$E_1 = 0.0654 \times 0.500 = 0.327$
$C_2 = 0.0359\ (1 - 0.385) = 0.0221$
$D_2 = 0.0359 \times 0.385 = 0.0138$
$E_2 = 0.0853 \times 0.500 = 0.0427$

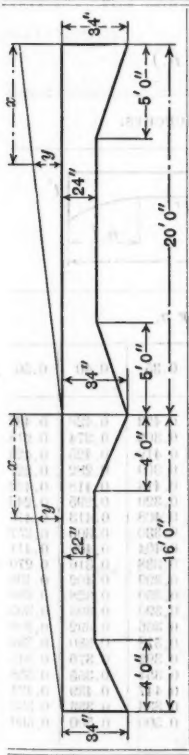
With these coefficients determined, Equation (9) may now be written for this problem as follows:

$$(0.0109\ M_1 + 0.0181\ M_2)\ l_1 + (0.0221\ M_2 + 0.0138\ M_3)\ l_2 = -\ (0.0327\ A_1 + 0.0427\ A_2)$$

or,

$$(M_1 + 1.66\ M_2)\ l_1 + (2.03\ M_2 + 1.27\ M_3)\ l_2 = -\ (3.00\ A_1 + 3.92\ A_2)$$

TABLE 1.—CALCULATION OF CONSTANTS FOR NUMERICAL EXAMPLE OF BEAM WITH VARYING I.



Depth, d.	$I = \frac{d^3}{1\,000}$.	x.	y.	$\frac{y}{I}$.	$\frac{y-x}{I}$.	y_a.	$\frac{y_a}{I}$.	$\frac{y_a - x}{I}$.
33	35.94	0.5	0.975	0.0271	0.014	0.007	0.0002	0.0001
31	29.79	1.5	0.925	0.0310	0.047	0.021	0.0007	0.0010
29	24.39	2.5	0.875	0.0359	0.090	0.033	0.0014	0.0034
27	19.68	3.5	0.825	0.0419	0.147	0.043	0.0022	0.0077
25	15.62	4.5	0.775	0.0496	0.223	0.052	0.0033	0.0151
24	13.82	5.5	0.725	0.0524	0.288	0.060	0.0043	0.0238
24	13.82	6.5	0.675	0.0488	0.317	0.066	0.0048	0.0309
24	13.82	7.5	0.625	0.0452	0.339	0.070	0.0051	0.0381
24	13.82	8.5	0.575	0.0416	0.354	0.073	0.0053	0.0451
24	13.82	9.5	0.525	0.0380	0.361	0.075	0.0054	0.0514
24	13.82	10.5	0.475	0.0344	0.361	0.075	0.0054	0.0568
24	13.82	11.5	0.425	0.0307	0.354	0.073	0.0053	0.0610
24	13.82	12.5	0.375	0.0271	0.339	0.070	0.0051	0.0636
24	13.82	13.5	0.325	0.0235	0.317	0.066	0.0048	0.0643
24	13.82	14.5	0.275	0.0199	0.288	0.060	0.0043	0.0627
25	15.62	15.5	0.225	0.0144	0.223	0.052	0.0033	0.0519
27	19.68	16.5	0.175	0.0089	0.147	0.043	0.0022	0.0363
29	24.39	17.5	0.125	0.0051	0.090	0.033	0.0014	0.0235
31	29.79	18.5	0.075	0.0025	0.047	0.021	0.0007	0.0129
33	35.94	19.5	0.025	0.0007	0.014	0.007	0.0002	0.0040
....	.....	.....	10.000	0.5787	4.360	1.000	0.0654	0.6536
32.5	34.33	0.5	0.969	0.0282	0.014	0.008	0.0002	0.0001
29.5	25.67	1.5	0.906	0.0353	0.053	0.023	0.0009	0.0014
26.5	18.61	2.5	0.844	0.0453	0.113	0.039	0.0021	0.0052
23.5	12.98	3.5	0.781	0.0602	0.211	0.055	0.0042	0.0147
22	10.65	4.5	0.719	0.0675	0.304	0.070	0.0066	0.0297
22	10.65	5.5	0.656	0.0616	0.339	0.086	0.0081	0.0448
22	10.65	6.5	0.594	0.0558	0.362	0.102	0.0096	0.0620
22	10.65	7.5	0.531	0.0499	0.374	0.117	0.0110	0.0825
22	10.65	8.5	0.469	0.0440	0.374	0.117	0.0110	0.0935
22	10.65	9.5	0.406	0.0382	0.362	0.102	0.0096	0.0906
22	10.65	10.5	0.344	0.0323	0.339	0.086	0.0081	0.0847
22	10.65	11.5	0.281	0.0264	0.304	0.070	0.0066	0.0759
23.5	12.98	12.5	0.219	0.0169	0.211	0.055	0.0042	0.0527
26.5	18.61	13.5	0.156	0.0084	0.113	0.039	0.0021	0.0283
29.5	25.67	14.5	0.094	0.0036	0.053	0.023	0.0009	0.0132
32.5	34.33	15.5	0.031	0.0009	0.014	0.008	0.0002	0.0035
....	.....	.....	8.000	0.5745	3.540	1.000	0.0854	0.6823

NOTE.—

$$m_1 = n_1 = \frac{0.5787}{20} = 0.0289;\ u_1 = v_1 = \frac{4.360}{20 \times 0.5787} = 0.376;\ f_1 = 0.0654;\ g_1 = \frac{0.6536}{20 \times 0.0654} = 0.500$$

$$m_2 = n_2 = \frac{0.5745}{16} = 0.0359;\ u_2 = v_2 = \frac{3.540}{16 \times 0.5745} = 0.385;\ f_2 = 0.0854;\ g_2 = \frac{0.6823}{16 \times 0.0854} = 0.500$$

Using either the coefficients of this last equation or the original factors, m, n, f, u, v, g, the principal points, U, V, T, are located as indicated in Fig. 23. The closing line is then easily obtained, as shown in Fig. 23, by applying the simplified construction of Section 10. The closing line passes through the outer U and V-points because the beam is assumed to be restrained at the ends.

14.—Use of Tabulated Constants for Cases of Varying I

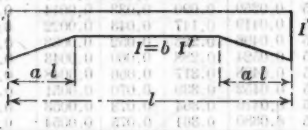
For practical use, tables may be constructed evaluating the constants, m, n, f, and u, v, g, for most common conditions, thus expediting the solution of problems of this type.

Table 2 is presented for this purpose, and gives the values of $m = n$ and $u = v$ for symmetrical beams with straight and parabolic haunches, respectively. Values for intermediate ratios not tabulated may be obtained by interpolation.

TABLE 2.—Constants for Beams of Varying I.

(Upper figures are values of $u = v$.
Lower figures, divided by I, give values of $m = n$.)

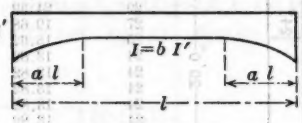
Straight Haunches.



Values of a.

	0.25	0.30	0.35	0.40	0.50
$b = 0.05$	0.425	0.434	0.441	0.444	0.434
	0.313	0.276	0.238	0.201	0.126
$b = 0.06$	0.422	0.431	0.437	0.440	0.430
	0.318	0.282	0.245	0.209	0.136
$b = 0.08$	0.416	0.425	0.430	0.431	0.422
	0.327	0.292	0.258	0.223	0.154
$b = 0.10$	0.412	0.419	0.424	0.425	0.416
	0.335	0.302	0.269	0.236	0.170
$b = 0.12$	0.407	0.415	0.419	0.420	0.410
	0.342	0.310	0.279	0.248	0.184
$b = 0.15$	0.402	0.408	0.412	0.412	0.403
	0.352	0.322	0.292	0.263	0.203
$b = 0.20$	0.395	0.400	0.402	0.402	0.394
	0.366	0.339	0.312	0.286	0.232
$b = 0.30$	0.382	0.385	0.387	0.387	0.380
	0.390	0.367	0.346	0.324	0.279
$b = 0.60$	0.356	0.360	0.358	0.357	0.359
	0.444	0.439	0.422	0.411	0.392
$b = 1.00$	0.333	0.333	0.333	0.333	0.333
	0.500	0.500	0.500	0.500	0.500

Parabolic Haunches.



Values of a.

	0.25	0.30	0.35	0.40	0.50
$b = 0.05$	0.406	0.415	0.423	0.428	0.433
	0.359	0.331	0.302	0.274	0.218
$b = 0.06$	0.408	0.412	0.419	0.425	0.429
	0.364	0.336	0.309	0.282	0.227
$b = 0.08$	0.398	0.407	0.413	0.418	0.422
	0.372	0.346	0.320	0.295	0.243
$b = 0.10$	0.395	0.403	0.408	0.413	0.416
	0.378	0.354	0.330	0.306	0.257
$b = 0.12$	0.392	0.398	0.404	0.408	0.411
	0.385	0.362	0.338	0.316	0.270
$b = 0.15$	0.387	0.393	0.399	0.402	0.405
	0.393	0.371	0.350	0.328	0.286
$b = 0.20$	0.381	0.386	0.390	0.393	0.395
	0.404	0.385	0.366	0.392	0.308
$b = 0.30$	0.371	0.375	0.378	0.380	0.382
	0.422	0.407	0.392	0.376	0.345
$b = 0.60$	0.351	0.353	0.354	0.355	0.355
	0.462	0.454	0.447	0.439	0.424
$b = 1.00$	0.333	0.333	0.333	0.333	0.333
	0.500	0.500	0.500	0.500	0.500

To illustrate the use of Table 2, it may be applied directly to the problem of the two-span beam, solved in Section 13. For the first span (having the ratio, $a = 0.25$, $b = 0.352$), Table 2 gives $u_1 = v_1 = 0.377$ (compared with 0.376 by the previous calculation) and $m_1 = n_1 = \dfrac{0.399}{13.82} = 0.0289$ (identical with the value previously calculated). For the second span (having the ratios, $a = 0.25$, $b = 0.27$), Table 2 gives $u_2 = v_2 = 0.386$ (compared with 0.385 by the previous calculation) and $m_2 = n_2 = \dfrac{0.383}{10.65} = 0.0359$ (identical with the value previously calculated).

Similar tables can easily be constructed to give the values of f and g for the common conditions of loading.

With such tables, the computations represented by Table 1 in the foregoing numerical example are dispensed with. The constants, m, n, f, and u, v, g, can be taken from the tables and used directly in the graphic solution shown in Figs. 22 and 23.

15.—Determination of Shears and Reactions

After the closing line, or M-line, is located for any problem by the foregoing methods, the shears and reactions are easily determined either graphically or analytically.

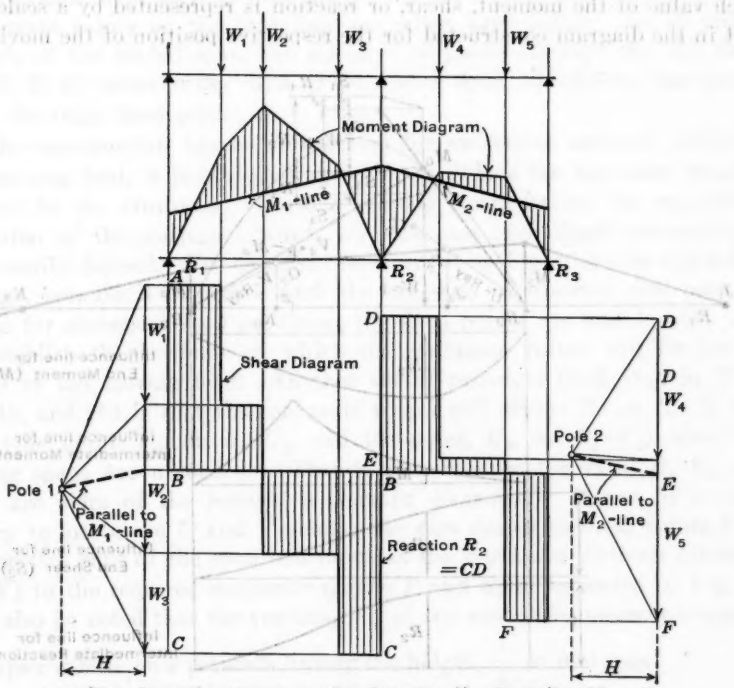


Fig. 24.—Determination of Shears and Reactions.

The graphic procedure is indicated in Fig. 24. In the pole diagram corresponding to the simple-beam moment graph of any span, a ray drawn parallel to the closing line divides the load line into two segments representing the respective end shears. The sum of the two end shears at any intermediate support gives the corresponding intermediate reaction. Horizontals from the division points of the load line intersect the load verticals in points of the desired shear diagram.

If the moment graph is curved, the outer rays of the pole diagram are drawn parallel to the terminal tangents in order to locate the pole.

It should be noted that the pole diagrams for successive spans of a continuous beam must have the same pole distance, H, because the respective moment graphs are drawn to the same vertical scale.

The foregoing construction is equivalent to the customary analytical procedure represented by the relation:

$$\text{Shear between } R_1 \text{ and } R_2 = \text{Shear for Simple Beam} + \frac{M_1 - M_2}{l_2}.$$

16.—Application to Construction of Influence Lines

If the foregoing methods are applied to a unit concentration assumed to be acting at successive points on a beam, the successive values of any moment, shear, or reaction can be plotted to produce the corresponding influence line. Each value of the moment, shear, or reaction is represented by a scaled intercept in the diagram constructed for the respective position of the moving load.

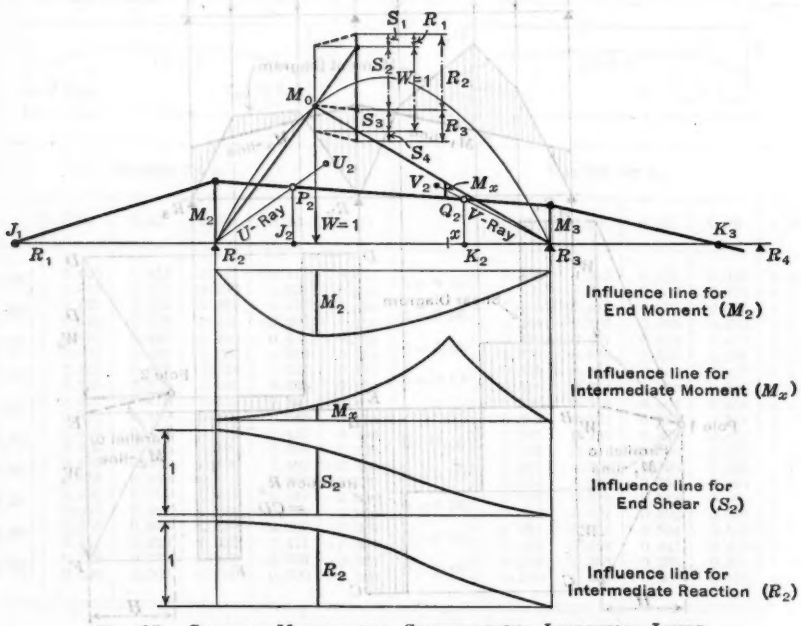


Fig. 25.—General Method for Constructing Influence Lines for Continuous Beams.

Fig. 25 shows the general procedure to be followed. A unit load, $w = 1$, is assumed to be acting at one of its successive positions on a continuous beam. The simple-beam moment graph is a triangle with a vertex, M_0, over the point of load; and the closing line is drawn through the conjugate points as indicated. The height of the closing line over any support indicates the value of the corresponding end moment, M_2 or M_3, and the vertical intercept between the M_0-triangle and the closing line at any intermediate section, X, gives the resultant moment, M_x, at the section. With the vertex, M_0, as a pole, a pole diagram is drawn with the unit load line, $w = 1$. A ray drawn parallel

to the closing line of the loaded span yields intercepts representing the end shears, S_2 and S_3. Rays drawn parallel to the respective directions of the closing line in the adjoining spans yield the shears, S_1 and S_4, in those spans. Summations of the shears for each support give the respective reactions, $R_1 = S_1$, $R_2 = S_1 + S_2$, and $R_3 = S_3 + S_4$.

The value of each moment, shear, or reaction is conveniently represented by a corresponding vertical intercept in the diagram. By plotting the successive values of one of these intercepts under successive positions of the moving load, the corresponding influence line is obtained. The influence lines obtained in this manner for an end moment, M_2, an intermediate moment, $M\,x$, an end shear, S_2, and an intermediate reaction, R_2, are shown in Fig. 25.

In the unloaded spans, the outer conjugate points are at zero height, that is, they coincide with their projections on the base line. The projections of the conjugate points on the base line are called fixed points. In all spans to the left of the loaded span, the closing line passes through the left fixed points, J; in all spans to the right of the loaded span, the closing line passes through the right fixed points, K.

As the construction has to be repeated for successive assumed positions of the moving load, it is obviously desirable to reduce the necessary graphic operations to the minimum. Various graphic constructions for expediting the location of the conjugate points for this case of a single concentrated load are easily devised. The principle represented by Fig. 9 can be applied to advantage, because it dispenses with the necessity of drawing new pennant diagrams for successive load positions; the fixed points are first located, and these establish the verticals on which the conjugate points will lie for all positions of the moving load. Another useful principle (indicated in Figs. 16, 17, 18, and 25) is that the conjugate points will always lie on the U and V-rays (the point, P_2, on $R_2\ U_2$, and the point, Q_2, on $R_3\ V_2$) when the adjoining spans are unloaded. (That is true because the rays, $R_2\ U_2$ and $R_3\ V_2$, are sides of the respective pennant diagrams.) Hence, it is only necessary to locate the U and V-points; the rays drawn to these points from the respective ends of the span will intersect the conjugate verticals (through J and K) in the required conjugate points, P and Q, as indicated in Fig. 25. It may also be noted that the vertices, M_0, of the successive moment triangles in any span will lie on a parabola having the height, $\frac{l}{4}$, at mid-span.

The detailed application of the foregoing to the graphic construction of influence lines for a continuous beam is shown in Fig. 26. Fig. 26 (a) shows the graphic location of the fixed points, J and K, which fix the verticals on which the conjugate points, P and Q, will lie. Fig. 26 (b), (c), and (d) show the graphic location of the closing line for a load position in the first, second, and third spans, respectively. In this construction, the G-line is marked at one-third the distance from the span center, C, to the load point, w; diagonals to the horizontal through M_0 intersect the G-line at the required heights of the U-V points; and rays to the U-V points intersect the conjugate verticals in the required conjugate points, P and Q, which locate the closing

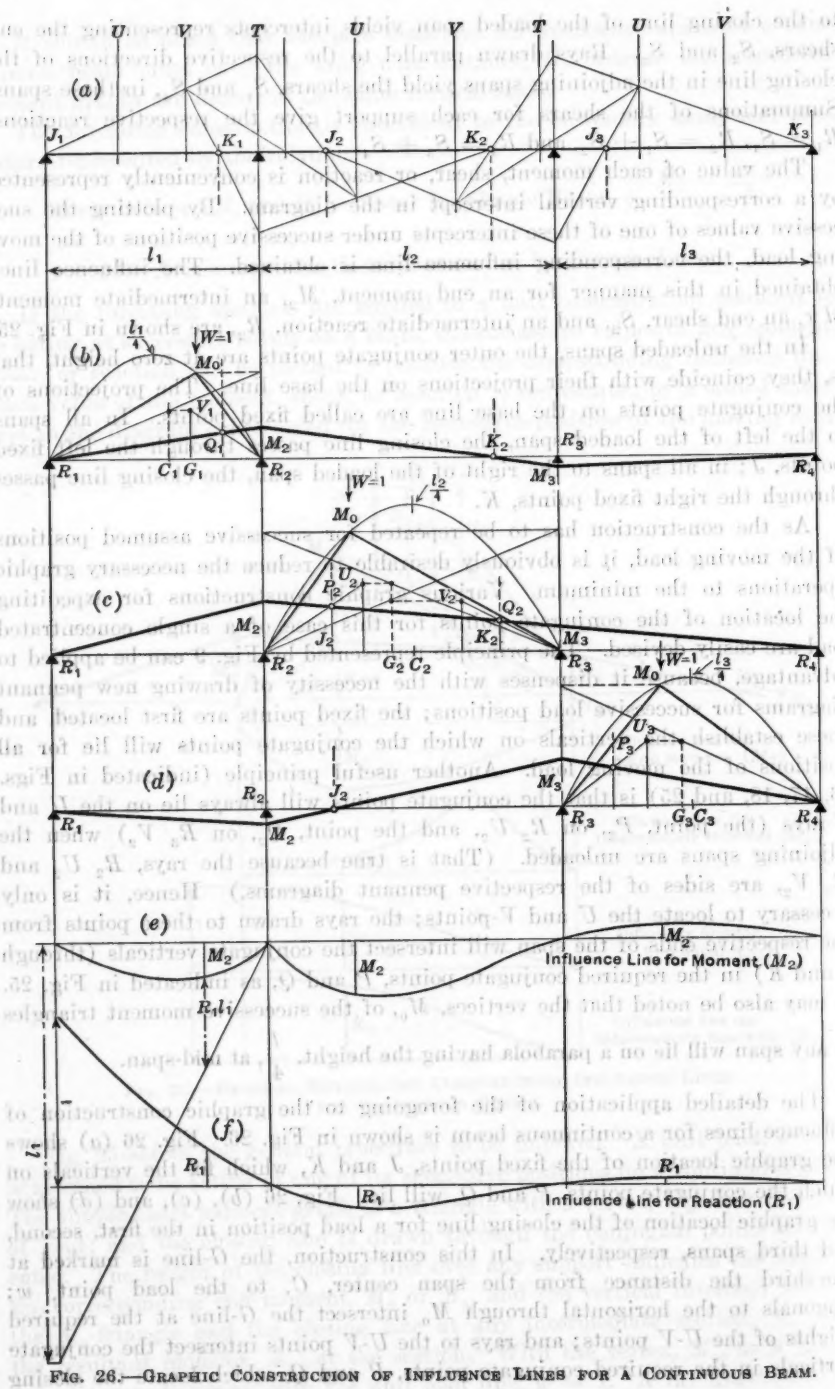


FIG. 26.—GRAPHIC CONSTRUCTION OF INFLUENCE LINES FOR A CONTINUOUS BEAM.

line. Fig. 26 (*e*) shows the influence line for the end moment, M_2, obtained by plotting successive values of the corresponding intercept over the support, R_2. The M_2 influence line is also the influence line for the end reaction, R_1 (if the ordinates are divided by l_1); except that in the span, l_1, the ordinates have to be measured from a different base line, namely, a diagonal having the end ordinate, l_1. Fig. 26 (*f*) shows the influence line thus obtained for R_1.

In Fig. 26, influence lines for other moments, shears, or reactions could easily be plotted from Fig. 26 (*b*), (*c*), and (*d*), using the respective intercepts as indicated in Fig. 25.

The influence diagram for shear or bending moment at any intermediate point of the end span, l_1, may also be obtained simply by adding a straight line to the R_1-influence line or "elastic curve" (Fig. 26 (*f*)).*

17.—Application to Settlement of Supports

The Method of Conjugate Points can also be applied advantageously to problems involving the settlement or deflection of one or more supports of a continuous or restrained beam.

If the right end of a span, l_1, settled more than the left end by an amount, d_1, the slope change for the span is represented by $\phi_1 = \frac{d_1}{l_1}$. Similarly, for the span, l_2, the slope change is represented by $\phi_2 = \frac{d_2}{l_2}$.

Introducing this condition into the analysis by which Equation (1) was written, that fundamental equation takes the form of Equation (13) for the case of settlement of supports:

$$(M_1 + 2\,M_2)\,l_1 + (M_3 + 2\,M_2)\,l_2 = -\,6\,E\,I\,(\phi_1 - \phi_2) \ldots\ldots (13)$$

Equation (13) expresses the Theorem of Three Moments for settlement of supports, for the case of constant I. For the case of unequal values of I, Equation (13) takes the following form, corresponding to Equation (8):

$$(M_1 + 2\,M_2)\,\frac{l_1}{I_1} + (M_3 + 2\,M_2)\,\frac{l_2}{I_2} = -\,6\,E\,(\phi_1 - \phi_2) \ldots\ldots (14)$$

It is observed that the left-hand members of Equations (13) and (14) are identical with those of Equations (1) and (8), respectively. The right-hand members, representing the influence of the deflections, replace the corresponding members of Equations (1) and (8) which represented the influence of the loading.

It is apparent, therefore, that the graphic constructions by the Method of Conjugate Points heretofore presented may also be applied to the present problem, except that the heights of the U, V, and T-points must be made to satisfy the changed form of the load terms in the equations. The locations of the U, V, and T-lines remain unchanged.

* See "Continuous Bridges" in Hool and Kinne's "Movable and Long-Span Steel Bridges", N. Y., 1923.

It is easily shown that both Equations (13) and (14) will be satisfied by the foregoing constructions, if the height of the T-point is made equal to:

$$T_{1-2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2} \qquad (15)$$

in which l_1 and e_2 are the "flexibility constants" of the respective spans, defined by $e_1 = \frac{l_1}{E\,I_1}$ and $e_2 = \frac{l_2}{E\,I_2}$. These constants are the reciprocals of the rigidity constants defined by $r_1 = \frac{E\,I_1}{l_1}$ and $r_2 = \frac{E\,I_2}{l_2}$.

If the I-values are unequal, the modified T-line is located (as established in Section 7) by dividing the distance between the adjacent U-V lines in the inverse ratio, $e_2 : e_1$, or in the direct ratio, $r_1 : r_2$.

Several constructions are available for applying Equation (15) for fixing the height of the T-points. The following methods are submitted:

1.—Locate the T-points by laying off the heights:

$$U = -2\,r\phi;\quad V = 2\,r\phi \qquad (16)$$

in each span, and then applying the usual construction. This is illustrated, in application to an actual problem, in Fig. 27. In order to show the respective treatments, one end of the continuous beam is assumed to be freely supported and the other end to be fixed. The values of the slope changes, ϕ, are scaled directly from the settlement diagram, as indicated.

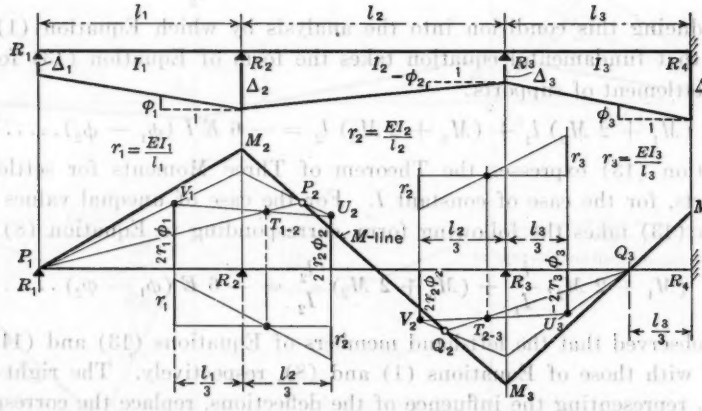


FIG. 27.—CONSTRUCTION OF M-LINE FOR SETTLEMENT OF SUPPORTS USING SLOPE CHANGES (ϕ).

2.—Locate each T-point directly by laying off its height as prescribed by Equation (15), namely,

$$T_{1-2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

This method is illustrated in Fig. 28. The values of the angle changes, $\phi_1 - \phi_2$, are scaled directly from the settlement diagram, as indicated.

3.—The device of transformed spans (as presented in Section 8) can be applied, using the values of U, V, and T defined by Equations (15) and (16) without change.

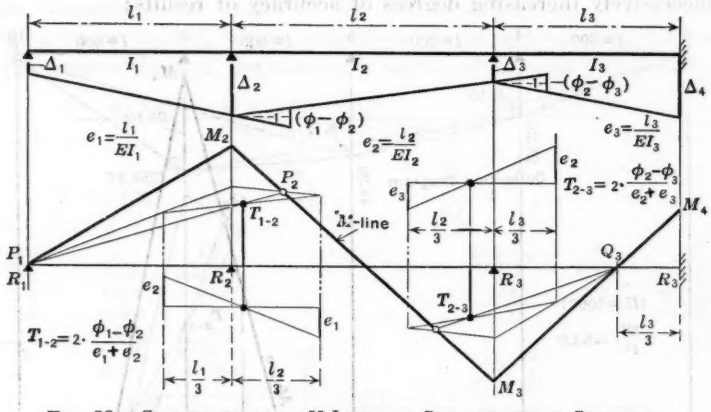


FIG. 28.—CONSTRUCTION OF M-LINE FOR SETTLEMENT OF SUPPORTS USING ANGLE CHANGES $(\phi_1 - \phi_2)$.

4.—If the successive spans are equal in length and in moment of inertia, the value of T reduces to:

$$T_{1-2} = \frac{2\,E\,I}{l^2}\,d_{1-2}$$

in which, d_{1-2} is the deflection of the support relative to the two adjoining supports. If straight lines are drawn joining the alternate supports, R_1 and R_3, in the settlement diagram, the distance of the intermediate support, R_2, below such chords is the value of the relative deflection, d_{1-2}. The heights of the T-points may be laid off equal to these values of d_{1-2}, d_{2-3}, etc., in which case the graphic results from the moments must be multiplied by the constant, $2\,\frac{E\,I}{l^2}$. (This method is illustrated in Fig. 29.)

18.—APPLICATION TO GRAPHIC METHODS FOR SECONDARY STRESSES

Bridge engineers have felt the need for practical methods of quickly ascertaining approximate values of secondary stresses in trusses. The standard methods of secondary stress analysis are, at the best, long and tedious; they require the formulation and solution of a long string of simultaneous equations, covering the moment relations at all the panel points of a structure, before the secondary stresses at a single selected point can be determined. The engineer, in practice, is satisfied to know the approximate magnitude of the secondary stresses at a few indicated points of a structure; the academic methods yield a degree of precision which is not warranted by the accuracy of the underlying assumptions nor by practical requirements.

The constructions and principles presented in the foregoing Sections can be applied in devising short-cut, graphic methods for the determination of

secondary stresses; the results will be sufficiently accurate for the usual requirements of the bridge engineer. Three methods for the graphic evaluation of secondary stresses by conjugate points will be presented that offer three successively increasing degrees of accuracy of results:

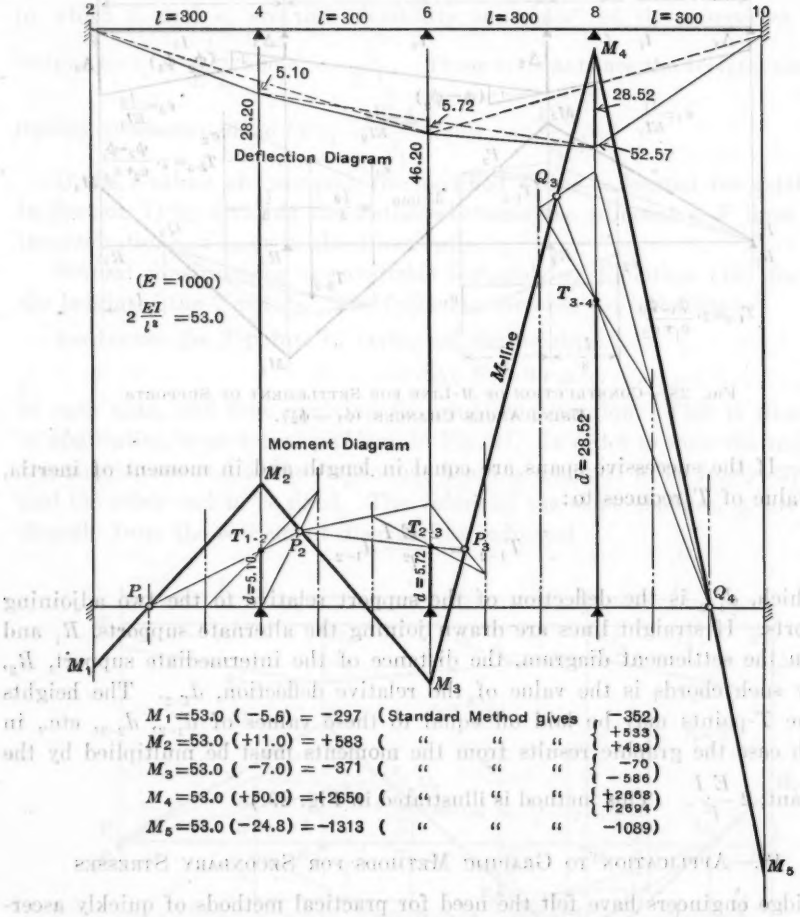


FIG. 29.—CONSTRUCTION FOR SECONDARY STRESSES, FIRST METHOD. (NEGLECTING WEB MEMBERS).

1.—The first method is an approximate method, neglecting the influence of members of comparatively small rigidity. It is illustrated in Fig. 29 in application to the top chord of a truss, with the influence of the web members regarded as negligible. The known vertical deflections of the panel points are plotted in a deflection diagram. As the successive members are equal in length and in moment of inertia, the construction of Section 17, Method 4, can be applied. The deflection of each intermediate panel point below the chord joining the adjacent panel points is scaled from the deflection diagram and is plotted as the height of the *T*-point on the corresponding reaction line

of the moment diagram. Drawing pennant diagrams through these T-points, the required conjugate points are located as shown, and the closing line drawn through these conjugate points represents the required secondary bending moments throughout the length of the chord. The ends of the chord are assumed to be restrained, which approximates the actual condition, so that the M-line passes through the one-third points of the end members.

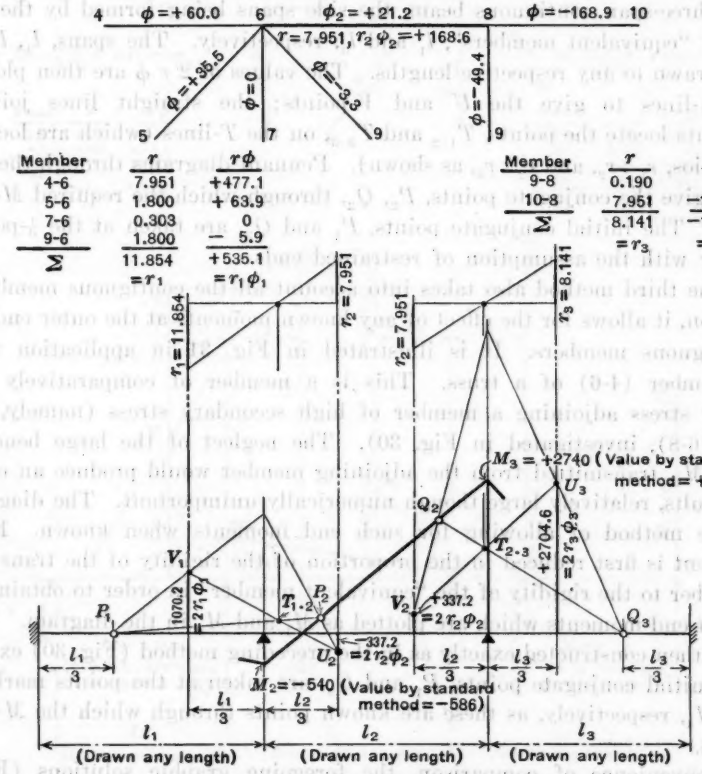


FIG. 30.—CONSTRUCTION FOR SECONDARY STRESSES, SECOND METHOD. (ALLOWING FOR WEB MEMBERS.)

2.—The second method takes into account the influence of the contiguous web members, but neglects the effect of the unknown moments at the outer ends of the contiguous members. It is based on a combination of the principles presented in Sections 7, 9, and 17. It is illustrated in Fig. 30 in application to a top chord member (6-8) of a truss. All truss members meeting the given member are taken into consideration. Their rigidities, $r = \frac{EI}{l}$ and their rotations, $\phi = \frac{d}{l}$ (determined from a Williot diagram which need not include more than the few members concerned) are noted in the diagram. The four members meeting the left end of the chord (6-8) are replaced by a single equivalent member, using the principle of the Composition of Rigidities;

the rigidity, r_1, of the equivalent member is the sum of the rigidities, r, of the component members, and the value of r_1, ϕ_1, for the equivalent member is the sum of the values of $r\,\phi$ for the component members. Similarly, the two members meeting the right end of the chord (6-8) are replaced by a single equivalent member of rigidity, r_3, and rotation, ϕ_3. The chord (6-8) itself has a rigidity, r_2, and a rotation, ϕ_2. This chord may now be considered as the middle span, l_2, of a three-span continuous beam, the side spans being formed by the left and right "equivalent members", l_1 and l_3, respectively. The spans, l_1, l_2, l_3, may be drawn to any respective lengths. The values of $2\,r\,\phi$ are then plotted on the $\frac{1}{3}$-lines to give the U and V-points; the straight lines joining these points locate the points, $T_{1\text{-}2}$ and $T_{2\text{-}3}$, on the T-lines (which are located by the ratios, $r_1 : r_2$, and $r_2 : r_3$, as shown). Pennant diagrams through the two T-points give the conjugate points, P_2, Q_2, through which the required M-line is drawn. The initial conjugate points, P_1 and Q_3, are taken at the $\frac{1}{3}$-points to comply with the assumption of restrained ends.

3.—The third method also takes into account all the contiguous members; in addition, it allows for the effect of any known moments at the outer ends of the contiguous members. It is illustrated in Fig. 31 in application to a chord member (4-6) of a truss. This is a member of comparatively low secondary stress adjoining a member of high secondary stress (namely, the member (6-8), investigated in Fig. 30). The neglect of the large bending moment, M_4, transmitted from the adjoining member would produce an error in the results, relatively large though numerically unimportant. The diagram shows the method of allowing for such end moments when known. Each end moment is first reduced in the proportion of the rigidity of the transmitting member to the rigidity of the "equivalent member" in order to obtain the equivalent end moments which are plotted as M_1 and M_4 in the diagram. The M-line is then constructed exactly as in the preceding method (Fig. 30) except that the initial conjugate points, P_1 and Q_3, are taken at the points marking M_1 and M_4, respectively, as these are known points through which the M-line must pass.

For convenience of comparison, the foregoing graphic solutions (Figs. 29, 30, 31) have been applied to the truss problem used by C. V. von Abo, Jun. Am. Soc. C. E., in his paper on "Secondary Stresses in Bridges."* The comparisons of the respective graphic results with those determined by Mr. von Abo, who used the standard methods, are noted in the diagrams and indicate the following conclusions.

1.—The first graphic method (Fig. 29) furnishes a good general idea of the character and magnitude of the secondary stresses in any selected chain of principal members in a truss. The errors, expressed as a percentage of the maximum secondary stress in the chain of members, range from 1 to 11 per cent.

2.—The second graphic method (Fig. 30) gives fairly accurate results, particularly if applied to those members of a truss where the secondary stresses

* *Proceedings*, Am. Soc. C. E., September, 1924, Papers and Discussions, p. 969.

are a maximum. The two secondary stresses found in Fig. 30 differ from the respective correct values by less than 8% and 3%; or by less than 2% and 3%, respectively, when expressed in percentage of the greater secondary stress.

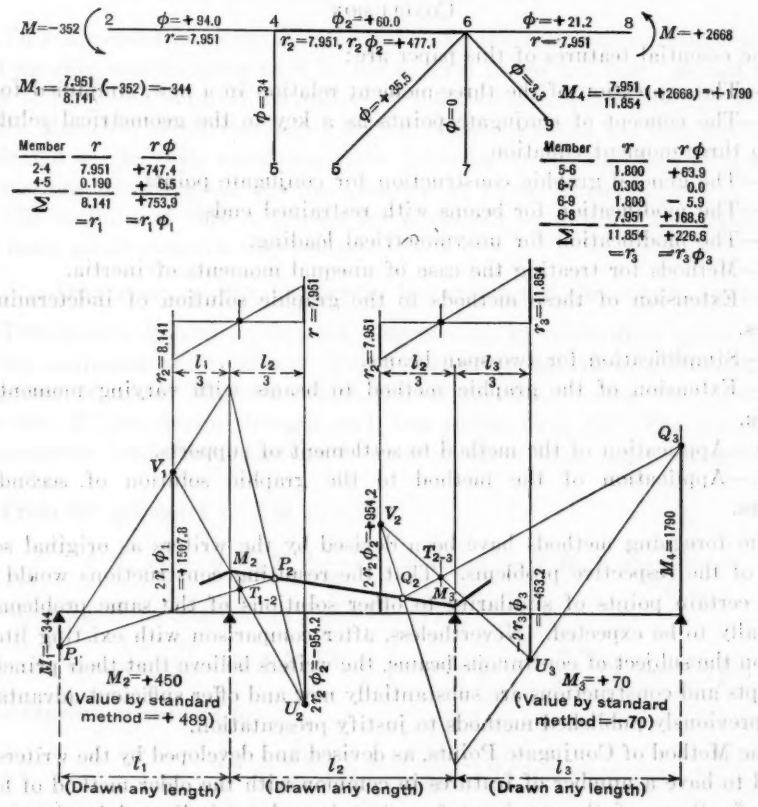


FIG. 31.—CONSTRUCTION FOR SECONDARY STRESSES, THIRD METHOD. (ALLOWING FOR WEB MEMBERS AND FOR END MOMENTS.)

3.—The third graphic method (Fig. 31) can be applied to yield results as accurate as may be desired, if the transmitted end moments are known. In Fig. 31, with the end moments transmitted by the contiguous chord members allowed for, but with the end moments transmitted by the web members neglected, the results differ from the correct values by only 1.5% and 5%, respectively, of the maximum secondary stress in the adjacent member.

In practical application, a member (such as 6-8, Fig. 30) in which maximum secondary stresses may be anticipated (from considerations of truss form and loading) would first be investigated by Method 2, and the results introduced as end moments for the investigation of successive adjoining members by Method 3. (Neglecting M_1 in Fig. 31 would not alter the results materially.)

With such procedure, fairly accurate values of the secondary stresses throughout the structure can be determined.

If approximate results suffice, the secondary stresses can be quickly estimated by Method 1 (Fig. 29).

Conclusion

The essential features of this paper are:

1.—The expression of the three-moment relation in a new, simplified form.

2.—The concept of conjugate points as a key to the geometrical solution of the three-moment equation.

3.—The general graphic construction for conjugate points.

4.—The modification for beams with restrained ends.

5.—The modification for unsymmetrical loading.

6.—Methods for treating the case of unequal moments of inertia.

7.—Extension of these methods to the graphic solution of indeterminate frames.

8.—Simplification for two-span beams.

9.—Extension of the graphic method to beams with varying moment of inertia.

10.—Application of the method to settlement of supports.

11.—Application of the method to the graphic solution of secondary stresses.

The foregoing methods have been devised by the writers as original solutions of the respective problems. That the resulting constructions would develop certain points of similarity to other solutions of the same problems is naturally to be expected. Nevertheless, after comparison with existing literature on the subject of continuous beams, the writers believe that their principal concepts and constructions are substantially new and offer sufficient advantages over previously published methods to justify presentation.

The Method of Conjugate Points, as devised and developed by the writers, is found to have a number of features in common with the older method of fixed points.* Some of these points of contact have been indicated in the paper. The two methods, however, are developed from different concepts; the procedure of fixed points has certain limitations in practical application; and the method of conjugate points is believed to be of greater flexibility and general usefulness. The latter method embraces the former as a special case.

The application of the concepts presented in this paper can be extended to the solution of many other problems of structural design. In the newer methods of stress analysis, there is a consistent trend toward simplification; the older, cumbersome, analytical procedures, frequently involving the solution of differential equations, are gradually being superseded by more attractive methods based on simple geometrical relations. As a contribution toward this objective, this paper is submitted to the profession.

* See "Continuous Bridges", in Hool and Kinne's "Movable and Long-Span Bridges", New York, 1923; also "Der kontinuierliche Balken", by Prof. W. Ritter, Zurich, 1900; and "Die Methode der Festpunkte", by Dr. Ernst Suter, Berlin, 1923.

APPENDIX

DEMONSTRATIONS OF THE FORMULAS AND CONSTRUCTIONS OF THE METHOD OF CONJUGATE POINTS

This Appendix gives the demonstrations of the principal working formulas and graphic constructions used in the paper. It has not been deemed necessary to include a detailed derivation of the special forms of the Equation of Three Moments, as the method of writing these equations has been sufficiently indicated in the text; moreover, these formulas are easily checked from the expressions usually given for the Theorem of Three Moments.

The sections of this Appendix are numbered to indicate the sections of the main paper to which they relate.

2.—Derivation of the Governing Equations for Conjugate Points

Two points, P_1 and P_2, located, respectively, in consecutive spans, l_1 and l_2, are conjugate points if any M-line drawn through them will satisfy the Equation of Three Moments which defines the true closing line. Consider any two M-lines drawn through such two points (Fig. 3). For the case of symmetrical loads and constant I, each of these lines must satisfy Equation (1).

From the geometry of Fig. 3:

$$\Delta M_1 = -\Delta M_2 \times \frac{l_1 - x_1}{x_1}\ ;\ \Delta M_3 = -\Delta M_2 \times \frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (1):

$$\left(-\Delta M_2 \times \frac{l_1 - x_1}{x_1} + 2\,\Delta M_2\right) l_1 + \left(-\Delta M_2 \times \frac{l_2 - x_2}{x_2} + 2\,\Delta M_2\right) l_2 = 0$$

This expression reduces to:

$$\left(3 - \frac{l_1}{x_1}\right) l_1 = \left(\frac{l_2}{x_2} - 3\right) l_2 \qquad (2)$$

which is the equation fixing the abscissa relationship of the two conjugate points, P_1 and P_2.

Also, from the geometry of Fig. 3:

$$M_1 = M_2 - (M_2 - y_1)\frac{l_1}{x_1} = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1}\, y_1$$

$$M_3 = M_2 - (M_2 - y_2)\frac{l_2}{x_2} = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2}\, y_2$$

Substituting these two relations in Equation (1), and then subtracting the expressions of Equation (2), there results:

$$\frac{l_1^2\, y_1}{x_1} + \frac{l_2^2\, y_2}{x_2} = -3\,(A_1 + A_2) \qquad (3)$$

which defines the ordinate relationship of the two conjugate points, P_1 and P_2, in terms of the moment areas, A_1 and A_2.

4.—Proof of the Construction for Conjugate Points

When any three points lie on the same straight line, the height of the intermediate point is the weighted mean of the heights of the two outer points, the weight factor for each height being the horizontal distance between the two other points. This principle of collinear points will be frequently utilized in the following demonstrations:

Applying this principle to the construction shown in Fig. 4 for locating the T-point, there results:

$$\left(\frac{l_1}{3}+\frac{l_2}{3}\right) T_{1-2} = V_1\left(\frac{l_1}{3}\right) + U_2\left(\frac{l_2}{3}\right)$$

whence,

$$T_{1-2} = \frac{V_1\, l_1 + U_2\, l_2}{l_1 + l_2} = \frac{A_1 + A_2}{l_1 + l_2} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The same result may also be written from the geometry of Fig. 4. Considering the triangles into which the diagram may be divided by the indicated dotted line, Fig. 4 yields:

$$T_{1-2} = \frac{l_1}{l_1 + l_2} V_1 + \frac{l_2}{l_1 + l_2} U_2 = \frac{A_1}{l_1 + l_2} + \frac{A_2}{l_1 + l_2} = \frac{A_1 + A_2}{l_1 + l_2} \ldots\ldots (5)$$

Equation (5) defines the height of the T-point as constructed.

Applying the same principle to the lines of Fig. 5, the following relations may be written:

As P_1, H_2, and B, are collinear:

$$h_1 = \frac{l_1}{3} \times \frac{y_1}{x_1} + \frac{b}{x_1}\left(x_1 - \frac{l_1}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{l_2}{3} \times \frac{y_2}{x_2} + \frac{b}{x_2}\left(x_2 - \frac{l_2}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

As H_1, T_{1-2}, and H_2, are collinear:

$$h_1\, l_1 + h_2\, l_2 = T_{1-2}\,(l_1 + l_2) \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$\frac{l_1^2}{3} \times \frac{y_1}{x_1} + \frac{b\, l_1}{x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2^2}{3} \times \frac{y_2}{x_2} + \frac{b\, l_2}{x_2}\left(x_2 - \frac{l_2}{3}\right) = T_{1-2}\,(l_1 + l_2) \ldots (d)$$

By Equation (2), the terms containing b in Equation (d) cancel, leaving:

$$\frac{l_1^2}{3} \times \frac{y_1}{x_1} + \frac{l_2^2}{3} \times \frac{y_2}{x_2} = T_{1-2}\,(l_1 + l_2) = A_1 + A_2 \ldots\ldots\ldots (e)$$

This agrees with Equation (3). Hence the construction of Fig. 5 satisfies Equations (2) and (3) which govern the location of the conjugate points.

The change in sign of the right-hand member of Equation (e) represents the reversal of the M-diagram about the X-axis, as explained in Section 4.

A special case of Equation (*d*) obtains when y_1, y_2, and T_{1-2}, are all zero. In that case, the pennant diagram drops to the base line as represented in Fig. 9. On substituting these zero values, Equation (*d*) reduces to:

$$\frac{l_1}{x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2}{x_2}\left(x_2 - \frac{l_2}{3}\right) = 0$$

which is identical with Equation (2). This is a check on the construction of Fig. 9.

If the right-hand member of Equation (1), representing the absolute or load-term of the Three-Moment Formula, is denoted by N, Equation (5) shows that the height of the T-point must be:

$$T_{1-2} = -\frac{N}{3\,(l_1 + l_2)} \qquad (f)$$

in order that the construction may satisfy the Equation of Three Moments. If the T-points are plotted in accordance with Equation (*f*), the foregoing pennant diagram construction will give the conjugate points, no matter what the value of N may be.

5.—Application to Beams with Restrained Ends

Applying the principle of collinear points to the construction of Fig. 12, if the M-line passes through U_1:

$$\frac{1}{3}M_2 + \frac{2}{3}M_1 = U_1 = \frac{A_1}{l_1}$$

This is identical with Equation (6); hence the construction of Fig. 12 satisfies Equation (6).

6.—Modification for Unsymmetrical Loading

By the construction of Fig. 14:

$$V_1 = \frac{g_1 l_1}{\frac{1}{2} l_1} \times \frac{A_1}{l_1} = 2\,g_1 \times \frac{A_1}{l_1} \qquad (a)$$

$$U_2 = \frac{g_2 l_2}{\frac{1}{2} l_2} \times \frac{A_2}{l_2} = 2\,g_2 \times \frac{A_2}{l_2} \qquad (b)$$

As V_1, T_{1-2}, and U_2, are collinear:

$$T_{1-2} = \frac{V_1 l_1 + U_2 l_2}{l_1 + l_2} \qquad (c)$$

Substituting Equations (*a*) and (*b*) in Equation (*c*):

$$T_{1-2} = 2\,\frac{g_1 A_1 + g_2 A_2}{l_1 + l_2} \qquad (d)$$

If the right-hand member of Equation (7) is denoted by N, Equation (*d*) yields:

$$T_{1-2} = -\frac{N}{3\,(l_1 + l_2)}$$

This relation is identical with Equation (*f*) of Section 4 of the Appendix; hence the Equation of Three Moments is satisfied.

7.—Modification for Unequal Moments of Inertia

The demonstrations for this case are parallel to those given in Sections 2 and 4 of the Appendix for the case of constant I.

The Equation of Three Moments for this case is Equation (8) instead of Equation (1). Consequently Equations (2) and (3), which govern the location of the conjugate points, are modified as follows: As before, from the geometry of Fig. 3:

$$\Delta M_1 = -\Delta M_2 \times \frac{l_1 - x_1}{x_1};\ \Delta M_3 = -\Delta M_2 \times \frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (8):

$$\left(-\Delta M_2 \times \frac{l_1 - x_1}{x_1} + 2\,\Delta M_2\right)\frac{l_1}{I_1} + \left(-\Delta M_2 \times \frac{l_2 - x_2}{x_2} + 2\,\Delta M_2\right)\frac{l_2}{I_2} = 0$$

This expression reduces to:

$$\left(3 - \frac{l_1}{x_1}\right)\frac{l_1}{I_1} = \left(\frac{l_2}{x_2} - 3\right)\frac{l_1}{I_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2')$$

which replaces Equation (2) for this case.

Also, from the geometry of Fig. 3:

$$M_1 = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1}\, y_1$$

$$M_3 = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2}\, y_2$$

Substituting these two relations in Equation (8) and then subtracting the expressions of Equation (2'), there results:

$$\frac{l_1^2\, y_1}{I_1\, x_1} + \frac{l_2^2\, y_2}{I_2\, x_2} = -3\left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3')$$

which replaces Equation (3) for this case.

Equations (2') and (3') are now to be satisfied, instead of Equations (2) and (3), by the graphic construction for conjugate points. The only change introduced in the construction is the modified location of the T-line as shown in Fig. 15.

With this modified location, the height of the T-point will be (by the principle of collinear points applied to V_1, $T_{1\text{-}2}$, and U_2):

$$T_{1-2} = \frac{V_1\left(\frac{l_1}{I_1}\right) + U_2\left(\frac{l_2}{I_2}\right)}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} = \frac{\frac{A_1}{I_1} + \frac{A_2}{I_2}}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5')$$

which corresponds to Equation (5).

The geometrical relations of Fig. 5 may now be applied to this new location of the T-point.

As P_1, H_1, and B, are collinear:

$$h_1 = \frac{l_1}{3} \times \frac{y_1}{x_1} + \frac{b}{x_1}\left(x_1 - \frac{l_1}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{l_2}{3} \times \frac{y_2}{x_2} + \frac{b}{x_2}\left(x_2 - \frac{l_2}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

As H_1, $T_{1\text{-}2}$, and H_2, are collinear:

$$h_1 \times \frac{l_1}{I_1} + h_2 \times \frac{l_2}{I_2} = T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) \ldots\ldots\ldots\ldots (c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$\frac{l_1^2}{3\,I_1} \times \frac{y_1}{x_1} + \frac{b\,l_1}{I_1\,x_1}\left(x_1 - \frac{l_1}{3}\right) + \frac{l_2^2}{3\,I_2} \times \frac{x_2}{x_2} + \frac{b\,l_2}{I_2\,x_2}\left(x_2 - \frac{l_2}{3}\right)$$

$$= T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

By Equation (2′), the terms containing b in this Equation (d) cancel, leaving:

$$\frac{l_1^2}{3\,I_1} \times \frac{y_1}{x_1} + \frac{l_2^2}{3\,I_2} \times \frac{y_2}{x_2} = T_{1-2}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) = \left(\frac{A_1}{I_1} + \frac{A_2}{I_2}\right) \ldots\ldots\ldots (e)$$

This agrees with Equation (3′). Hence the construction of Fig. 5, applied with the T-point located in accordance with Fig. 15, satisfies Equations (2′) and (3′) which prescribe the location of the conjugate points.

When $I_1 = I_2$, all the foregoing equations reduce to the simpler equations of Sections 2 and 4.

12.—Application to Beams with Varying Moment of Inertia

For this case, the Equation of Three Moments assumes its most general form, namely:

$$(C_1\,M_1 + D_1\,M_2)\,l_1 + (C_2\,M_2 + D_2\,M_3)\,l_2 = -\,(E_1\,A_1 + E_2\,A_2) \ldots (12)$$

The following demonstrations pertaining to the graphic solution of Equation (12) are parallel to the demonstrations given in connection with Equations (1) and (8). (Compare Sections 2, 4, and 7 of the Appendix.)

From the geometry of Fig. 3:

$$\Delta\,M_1 = -\,\Delta\,M_2\,\frac{l_1 - x_1}{x_1};\quad \Delta\,M_3 = -\,\Delta\,M_2\,\frac{l_2 - x_2}{x_2}$$

Substituting these relations in Equation (12), there results:

$$\left(-\,C_1\,\frac{l_1 - x_1}{x_1} + D_1\right)l_1 + \left(C_2 - D_2\,\frac{l_2 - x_2}{x_2}\right)l_2 = 0$$

This expression reduces to:

$$\left(C_1 + D_1 - \frac{l_1}{x_1}\,C_1\right)l_1 = -\left(C_2 + D_2 - \frac{l_2}{x_2}\,D_2\right)l_2 \ldots\ldots\ldots (2'')$$

which replaces Equation (2) as the general equation fixing the abscissa-relationship of the two conjugate points, P_1 and P_2.

Also, from the geometry of Fig. 3:

$$M_1 = \left(1 - \frac{l_1}{x_1}\right) M_2 + \frac{l_1}{x_1} y_1$$

$$M_3 = \left(1 - \frac{l_2}{x_2}\right) M_2 + \frac{l_2}{x_2} y_2$$

Substituting these two relations in Equation (12) and then subtracting the expressions of Equation (2″), there results:

$$C_1 \frac{l_1^2 y_1}{x_1} + D_2 \frac{l_2^2 y_2}{x_2} = (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots\ldots (3'')$$

which replaces Equation (3) as the general equation defining the ordinate relationship of the two conjugate points, P_1 and P_2.

Equations (2″) and (3″) are now to be satisfied, instead of Equations (2) and (3), by the graphic construction for conjugate points. The only changes introduced in the construction are in the more generalized locations of the U, V, and T-points as shown in Fig. 22.

With these modified locations, the heights of the U-V points are defined by:

$$V_1 = \frac{F_1}{C_1 + D_1} \times \frac{A_1}{l_1};\ U_2 = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2} \ldots\ldots\ldots\ldots (4'')$$

and the resulting height of the T-point will be (by the principle of collinear points applied to V_1, $T_{1\text{-}2}$, and U_2):

$$T_{1\text{-}2} = \frac{(C_1 D_1)\, l_1 \times V_1 + (C_2 + D_2)\, l_2 \times U_2}{(C_1 + D_1)\, l_1 + (C_2 + D_2)\, l_2}$$

$$= \frac{E_1 A_1 + E_2 A_2}{(C_1 + D_1)\, l_1 + (C_2 + D_2)\, l_2} \ldots\ldots\ldots\ldots\ldots (5'')$$

which is a generalized form of Equation (5).

The geometrical relations of Fig. 5 may now be applied to these new locations of the T-point and the U-V lines.

As P_1, H_1, and B, are collinear:

$$h_1 = \frac{C_1}{C_1 + D_1} \times \frac{l_1}{x_1} \times y_1 + b \left(1 - \frac{C_1}{C_1 + D_1} \times \frac{l_1}{x_1}\right) \ldots\ldots\ldots (a)$$

As B, P_2, and H_2, are collinear:

$$h_2 = \frac{C_2}{C_2 + D_2} \times \frac{l_2}{x_2} \times y_2 + b \left(1 - \frac{D_2}{C_2 + D_2} \times \frac{l_2}{x_2}\right) \ldots\ldots\ldots (b)$$

As H_1, $T_{1\text{-}2}$, and H_2, are collinear:

$$h_1 (C_1 + D_1)\, l_1 + h_2 (C_2 + D_2)\, l_2 = T_{1\text{-}2} [(C_1 + D_1)\, l_1 + (C_2 + D_2)\, l_2] \ldots (c)$$

Substituting the values of h_1 and h_2 from Equations (a) and (b) in Equation (c):

$$C_1 l_1^2 \frac{x_1}{y_1} + b \left[(C_1 + D_1)\, l_1 - C_1 \frac{l_1^2}{x_1}\right] + C_2 l_2^2 \frac{x_2}{y_2}$$

$$+ b \left[(C_2 + D_2)\, l_2 - D_2 \frac{l_2^2}{x_2}\right] = T_{1\text{-}2} [(C_1 + D_1)\, l_1 + (C_2 + D_2)\, l_2]$$

$$= (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots\ldots\ldots (d)$$

By Equation (2″), the terms containing b in this Equation (d) cancel, leaving:

$$C_1 l_1^2 \frac{y_1}{x_1} + C_2 l_2^2 \frac{y_2}{x_2} = (E_1 A_1 + E_2 A_2) \ldots\ldots\ldots\ldots\ldots (e)$$

This agrees with Equation (3″). Therefore, the construction of Fig. 5, applied with the U-V lines and the T-point located in accordance with Fig. 22, satisfies the generalized Equations (2″) and (3″) which prescribe the location of the conjugate points.

Instead of using the equational coefficients, C, D, and E, the beam constants, m, n, u, and v, and the load constants, f, g, may be used for plotting the U, V, and T-points.

By definition:

$$m = \frac{1}{l} \int \frac{l - x}{I\, l}\, d\, x = \frac{1}{l^2} \int \frac{l - x}{I}\, d\, x$$

$$n = \frac{1}{l} \int \frac{x}{I\, l}\, d\, x = \frac{1}{l^2} \int \frac{x}{I}\, d\, x$$

$$u = \frac{\int \frac{x\,(l - x)}{I\, l}\, d\, x}{l \int \frac{l - x}{I\, l}\, d\, x} = \frac{1}{m\, l^3} \int \frac{x\,(l - x)}{I}\, d\, x$$

$$v = \frac{\int \frac{x\,(l - x)}{I\, l}\, d\, x}{l \int \frac{x}{I\, l}\, d\, x} = \frac{1}{n\, l^3} \int \frac{x\,(l - x)}{I}\, d\, x = \frac{m}{n}\, u$$

$$f = \frac{\int \frac{y}{I}\, dx}{\int y\, dx} = \frac{1}{A} \int \frac{y}{I}\, dx$$

$$g = \frac{\int \frac{x\, y}{I}\, d\, x}{l \int \frac{y}{I}\, d\, x} = \frac{1}{A\, f\, l} \int \frac{x\, y}{I}\, d\, x$$

in which, y denotes the ordinates of the simple-beam moment area, and the integrations extend over the length of the span.

Hence, by definition:

$$C_1 = m_1 u_1 = \frac{1}{l_1^3} \int_0^{l_1} \frac{x\,(l_1 - x)}{I_1}\, d\, x = n_1 v_1$$

$$D_1 = n_1\,(1 - v_1) = n_1 - C_1$$

Therefore,

$$C_1 + D_1 = n_1$$

$$E_1 = f_1 g_1 = \frac{1}{A_1 l_1} \int_0^{l_1} \frac{x\, y_1}{I_1}\, d\, x$$

and,

$$D_2 = n_2 v_2 = \frac{1}{l_2^3}\int_0^{l_2} \frac{x\,(l_2 - x)}{I_2}\,d\,x = m_2 u_2$$

$$C_2 = m_2\,(1 - u_2) = m_2 - D_2$$

Therefore,

$$C_2 + D_2 = m_2$$

$$E_2 = f_2\,(1 - g_2) = \frac{1}{A_2\,l_2}\int_0^{l_2} \frac{(l - x)\,y_2}{I_2}\,d\,x$$

From these expressions, the following simple relations may be written:

$$\frac{C_1}{C_1 + D_1} = v_1\ ;\ \frac{D_1}{C_1 + D_1} = 1 - v_1$$

$$\frac{C_2}{C_2 + D_2} = 1 - u_2\ ;\ \frac{D_2}{C_2 + D_2} = u_2\ .$$

$$\frac{E_1}{C_1 + D_1} = \frac{f_1\,g_1}{n_1}\ ;\ \frac{E_2}{C_2 + D_2} = \frac{f_2\,(1 - g_2)}{m_2}$$

These relations explain the alternative values indicated in Fig. 22 for locating the U, V, and T-points.

For unsymmetrical conditions, Fig. 22 indicates how the U-V points may be located from the O-point by making the height of the latter:

$$O = \frac{f}{4\,m\,u} \times \frac{A}{l} = \frac{f}{4\,n\,v} \times \frac{A}{l}$$

The resulting heights of the U-V points, by this construction, are:

$$V_1 = \frac{v_1\,l_1}{\frac{1}{2}\,l_1} \times \frac{g_1\,l_1}{\frac{1}{2}\,l_1} \times \frac{f_1}{4\,n_1\,v_1} \times \frac{A_1}{l_1} = \frac{f_1\,g_1}{n_1} \times \frac{A_1}{l_1} = \frac{E_1}{C_1 + D_1} \times \frac{A_1}{l_1}$$

$$U_2 = \frac{u_2\,l_2}{\frac{1}{2}\,l_2} \times \frac{(1 - g_2)\,l_2}{\frac{1}{2}\,l_2} \times \frac{f_2}{4\,m_2\,u_2} \times \frac{A_2}{l_2}$$

$$= \frac{f_2\,(1 - g_2)}{m_2} \times \frac{A_2}{l_2} = \frac{E_2}{C_2 + D_2} \times \frac{A_2}{l_2}$$

These values agree with Equation (4″), which is a check on the indicated construction.

17.—Application to Settlement of Supports

It was shown in Section 4 of the Appendix that if N denotes the right-hand member of a Three-Moment Equation for constant I, the equation will be satisfied by the conjugate point construction if:

$$T_{1\text{-}2} = -\frac{N}{3\,(l_1 + l_2)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (f)$$

For settlement of supports, the Theorem of Three Moments is expressed by Equation (13); consequently, for this case:

$$N = -6\,E\,I\,(\phi_1 - \phi_2)$$

Hence, the necessary height of the T-point is:

$$T_{1-2} = 2\,E\,I\frac{\phi_1 - \phi_2}{l_1 + l_2} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which checks Equation (15).

For unequal values of I, it is shown in Section 7 of the Appendix (comparing Equation (5′) with Equation (8)) that the necessary height of the T-point is:

$$T_{1-2} = -\frac{N}{3\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f')$$

For this case (settlement of supports, with unequal I), Equation (14) gives:

$$N = -6\,E\,(\phi_1 - \phi_2)$$

Substituting this value in Equation (f'), there results:

$$T_{1-2} = 2\,E\,\frac{\phi_1 - \phi_2}{\frac{l_1}{I_1} + \frac{l_2}{I_2}} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which again checks Equation (15).

The location of the T-line for settlement of supports is conveniently accomplished by applying the ratio, $\frac{r_1}{r_2} = \frac{e_2}{e_1}$, as shown in Figs. 27 and 28. If E is constant, this ratio is identical with the ratio, $\frac{l_2}{I_2} : \frac{l_1}{I_1}$, applied in Fig. 15; and if I is also constant, the latter ratio reduces to $l_2 : l_1$, applied in Fig. 4.

If the construction shown in Fig. 27 is used,

$$V_1 = 2\,r_1\,\phi_1;\; U_2 = -2\,r_2\,\phi_2.$$

As V_1, T_{1-2}, and U_2, are collinear:

$$T_{1-2} = \frac{2\,r_1\,\phi_1 \times r_2 - 2\,r_2\,\phi_2 \times r_1}{r_1 + r_2} = 2\,\frac{\phi_1 + \phi_2}{\frac{1}{r_1} + \frac{1}{r_2}} = 2\,\frac{\phi_1 - \phi_2}{e_1 + e_2}$$

which satisfies Equation (15).

RELATION OF THE OHIO RIVER AND ITS TRIBUTARIES TO TRANSPORTATION IN THE UNITED STATES

A SYMPOSIUM*

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THE RELATION OF THE OHIO RIVER AND ITS TRIBUTARIES TO TRANSPORTATION IN THE UNITED STATES

By C. W. Kutz,* M. Am. Soc. C. E.

The Ohio River extends from the confluence of the Allegheny and the Monongahela Rivers at Pittsburgh, Pa., to the Mississippi River at Cairo, Ill., a distance of 968 miles. Its width varies from less than 900 ft. to nearly 6 000 ft. and its slope from 11½ in. per mile in the upper reach, to 3 in. per mile near the mouth.

The more or less uniformly changing slope has one break at the Falls of the Ohio (Louisville, Ky.), where there is a low-water drop of 26 ft. in a distance of 2 miles. The low-water discharge varies from 1 100 sec-ft. at Pittsburgh, where the water-shed is 18 970 sq. miles, to 27 500 sec-ft. at the mouth, where the water-shed is 203 900 sq. miles, and the maximum high-water discharge (estimated) varies from 440 000 sec-ft. at Pittsburgh to 1 500 000 sec-ft. at Cairo. The extreme fluctuations of stage at different points along the river due to floods, and the year in which they occurred, are indicated in Table 1.

TABLE 1.

Place.	Highest recorded stage above low water, in feet.	Year.	Place.	Highest recorded stage above low water, in feet.	Year.
Pittsburgh, Pa.	35.5	1907	Portsmouth, Ohio	67.9	1913
Beaver, Pa.	47.1	1907	Cincinnati, Ohio	71.1	1884
Steubenville, Ohio	48.6	1884	Louisville, Ky. (below Falls)	70.0	1884
Wheeling, W. Va.	52.1	1884	Paducah, Ky.	54.3	1884
Parkersburg, W. Va.	58.9	1913	Cairo, Ill.	54.8	1913
Point Pleasant, W. Va.	68.8	1913			

In its original condition, the river, in addition to the Falls at Louisville, was obstructed by snags, rocks, and bars composed of sand and gravel. The minimum depth over bars at extreme low water was about 1 ft. in the upper, and about 2 ft. in the lower, section. In some of the intervening pools the depths were as great as 30 ft. In 1825, just 100 years ago, the Legislature of Kentucky empowered a stock company to construct a canal around the Falls of the Ohio with suitable locks, docks, and basin. It was completed in 1830. In addition to a canal 64 ft. wide, there was a flight of three locks of equal lifts at the western end, each chamber having a width of 50 ft. and a usable length of 185 ft. The Federal Government became a stockholder in this company in 1826. Tolls were charged and the project was a profitable one. In 1874, the Government acquired complete title and tolls were reduced from 50 cents to 5 cents per ton. A few years later it was made a free waterway.

* Col., Corps of Engrs., U. S. A., Cincinnati, Ohio.

Until 1879 work elsewhere on the river was limited to the removal of obstructions and the building of contracting dikes with a view to securing a low-water depth of 3 ft. In this year Congress authorized the construction of a lock and dam about 5 miles below Pittsburgh, which proved to be the beginning of a slack-water system for the entire river. As first proposed the canalization project contemplated a depth of only 6 ft. and covered a distance of only 90 miles. Even as late as 1896, it was limited to the reach above Marietta, Ohio, 174 miles in length.

As now formulated the project provides for a navigable depth of 9 ft. throughout by the construction of fifty low-lift dams, each with a lock chamber 110 ft. wide and 600 ft. long in the clear. To prevent any increase in flood height and to permit open river navigation when the natural depth is 9 ft. or more, all the dams are of the movable type except the one immediately below Pittsburgh. This is of recent construction and replaced Dams No. 1 and No. 2 as originally built.

The main part of each movable dam, through which navigation passes when the dam is lowered, is known as the navigable pass. It is from 600 to 1 000 ft. long and consists of a series of wickets or shutters, 3 ft. 9 in. wide and 16 to 20 ft. long, depending on the lift of the dam, each supported in an inclined position by a prop. The dam is lowered by means of a derrick-boat, known as a maneuver-boat, the props being released from their seats one by one and the wickets lowered so as to lie flush with the bottom of the river. In addition to the lock and navigable pass there are weirs of different types for the purpose of regulating the pools and facilitating the raising and lowering of the navigable pass.

Of the fifty dams, thirty-eight are completed and in operation, and the remaining twelve are under construction. Of these, four are nearly completed and on two work of a preliminary character only has been done. The reach from Pittsburgh to Cincinnati, Ohio, will be completed in 1925 and the last dams should be completed in the season of 1929. This relatively long construction period is due to the fact that the various parts of a lock and dam are built within coffer-dams, designed to hold out the river to a stage of 16 ft. When the river rises above this stage, work must be suspended until the water subsides. With this limitation, the working season rarely exceeds 5 months—July to November, inclusive—and sometimes is as short as 100 working days. Under normal conditions, 4 years are required for the construction of a lock and dam, and, in the lower river where the dams are longer, 5 years will be required.

Tributaries.—In Table 2 are listed the principal navigable tributaries of the Ohio with information that tends to show their relative magnitude and the nature of the improvements made to date. Navigable sections of those rivers not now connected with the Ohio are not included in Table 2. An exception to this rule is made in the case of the Cumberland River, the canalized part of which will not have a 12-months connection with the Ohio until Dam No. 52, Ohio River, is completed. On the Allegheny, Big Sandy, Cumberland,

and Tennessee Rivers, where authorized projects are incomplete, only the completed sections at the lower end are given.

The lack of uniformity in lock dimensions is partly accounted for by the fact that on five of these tributaries, the canalization projects were first undertaken by the States or by private capital and subsequently acquired or taken over by the Federal Government, but even where not thus hampered, there was an apparent failure on the part of the Government to standardize lock dimensions to the extent that now seems desirable.

TABLE 2.—PRINCIPAL NAVIGABLE TRIBUTARIES OF THE OHIO RIVER.

Mouth, in miles below Pittsburgh.	Name.	Drainage area, in square miles.	Low-water discharge, in second-feet.	Length canalized, in miles.	Number of dams.	Project depth, in feet, canalized section.	Dimension of locks, in feet.	Notes.
0	Allegheny	11 580	1 440	24	3	7	55 by 286	Locks Nos. 2 and 3 slightly larger.
0	Monongahela	7 390	160	101 30	8 6	8 7	56 by 360 56 by 182	This assumes completion of new Locks and Dams Nos. 7 and 8 now being built to replace old Nos. 7, 8, and 9. Locks Nos. 1 to 6 have double lock chambers. At Lock No. 3 one of the lock chambers is 720 ft. long.
172	Muskingum	7 797	672	91	11	5.5	36 by 160	Lock No. 1 is 56 by 360 ft.
184	Little Kanawha	2 288	....	48	5	4	22.7 by 125	One lock is 26 ft. wide.
265	Kanawha	12 200	1 100	88	6 4	6 6	55 by 313 50 by 271	.....
317	Big Sandy	4 182	100	55	5	6	55 by 158	Lock No. 3 is only 52 ft. wide. Three locks on main river and one on each fork.
543	Kentucky	5 904	100	270	14	6	37 by 145 52 by 148	.....
779	Green	13 000	....	217	7	5.5	35.5 by 137.5	One of the locks is on Barren River, a tributary.
910	Cumberland	17 758	1 100	333	14	6	52 by 280	.....
924	Tennessee	40 569	10 000	256	*	6	80 by 287	Florence, Ala., to mouth only.

* One lock and lateral canal, 8 miles long.

Table 3 shows the first cost of the existing river improvement projects in the Ohio Basin and the cost of operation and maintenance. In the case of the Ohio River, the amount given is 80% actual cost and 20% estimated. On the Monongahela River, the figure includes the estimated cost of Locks and Dams Nos. 7 and 8, now under construction, to replace old Locks and Dams Nos. 7, 8, and 9. In other cases the cost is the historical cost as shown by the Annual Reports of the Chief of Engineers, U. S. Army. The costs of operation and maintenance are compiled in a similar manner. In Column (4) of Table 3 is shown the cost per mile and, in Column (5), interest on the first cost at 4 per cent. Column (7) shows the total fixed charges and Column (8) the fixed charges per mile.

Commerce.—Table 4 shows, for the Ohio and for the tributaries listed, the commerce for certain selected years as taken from the Annual Reports of the

Chief of Engineers, U. S. Army. The year 1900 was selected as marking the peak of water-borne commerce on the Ohio, and 1913 as typical of conditions just prior to the World War. Present conditions are shown by the tonnage reported for the calendar year 1924, and by the average for the period 1919-24, inclusive. The approximate ton-mileage in 1924, the fixed charges per ton-mile, and the cost of maintenance and operation per ton-mile are also shown.

TABLE 3.—OHIO RIVER AND THE CANALIZED PARTS OF ITS PRINCIPAL TRIBUTARIES.

River.	Length, in miles.	Cost.	Cost per mile.	Interest on first cost at 4%.	Cost of operation and maintenance average for 5 years.	Fixed charges Column (5) + Column (6).	Fixed charges per mile.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ohio	968	$103 000 000	$106 350	$4 120 000	$2 000 000	$6 120 000	$6 320
Allegheny	24	1 446 000	60 200	57 840	136 080	193 920	8 075
Monongahela	131	13 612 000	103 900	544 480	1 185 500	1 729 980	13 200
Muskingum	91	1 452 595	15 963	58 104	76 670	144 774	1 591
Little Kanawha	48	282 000	5 800	11 280	29 857	41 137	857
Kanawha	88	3 580 000	40 700	143 200	166 160	309 360	3 515
Big Sandy	55	1 512 000	26 500	60 480	34 707	95 187	1 731
Kentucky	260	4 266 000	16 400	170 640	234 541	405 181	1 558
Green	217	670 428	3 100	26 817	102 770	129 587	597
Cumberland, from mouth to Lock and Dam No. 9	333	7 174 000	21 544	286 960	105 036	391 996	1 177
Tennessee below Florence, Ala	256	5 166 000	20 180	206 640	45 000	251 640	983

TABLE 4.—COMMERCE ON THE OHIO RIVER AND ON PARTS OF ITS TRIBUTARIES DIRECTLY CONNECTED THEREWITH (EXCLUSIVE OF FLOATED TIMBER).

River.	Tonnage, 1900.	Tonnage, 1913.	Tonnage, 1924.	Average tonnage, 1919-24, inclusive.	Ton-mileage, 1924.	Fixed charges, in mills per ton-mile.	Cost of maintenance and operation, mills per ton-mile.
Ohio	14 054 000	9 814 000	8 446 322	7 416 119	978 421 930	6.25	2.04
Allegheny	1 611 103	2 159 530	4 339 511	3 940 686	35 370 300	5.5	3.85
Monongahela	7 000 000	12 039 175	21 878 815	19 558 108	919 564 942	1.88	1.29
Muskingum	78 595	61 755	152 012	120 667	912 000	16.1	8.4
Little Kanawha	119 439	97 099	45 951	40 907	1 378 530	29.7	21.6
Kanawha	1 475 930	1 416 804	901 069	1 146 395	54 064 140	5.7	3.18
Big Sandy	300 000	172 403	29 497	55 274	450 780	211.0	77.0
Kentucky	106 336	211 794	195 372	177 475	40 521 667	10.0	5.9
Green	1 190 756	1 057 063	486 916	313 206	*	.....	.....
Cumberland	696 306	626 832	390 700	358 846	15 949 200	24.5	6.6
Tennessee, below Florence, Ala	.....	584 191	1 255 436	558 000	59 202 500	4.25	0.76

*Not available.

Strictly speaking this commerce is not all applicable to the sections of the rivers listed in the Table 4, as, in some cases where separation was impossible, it includes all the commerce reported for the entire river, but the error thus introduced is not large. The tonnage reported for recent years is exclusive of floated timber and timber products. For earlier years such tonnage was not

separately reported. The amounts listed include sand and gravel much of which is transported for very short distances. To show the nature of the traffic, the reported tonnage for 1924 has been divided into four groups: Coal, sand and gravel, other bulk freight, and package freight, as given in Table 5.

TABLE 5.—REPORTED TONNAGE OF FOUR COMMODITIES ON THE OHIO RIVER AND ON PARTS OF ITS TRIBUTARIES IN 1924.

River.	Coal.	Sand and gravel.	Other bulk freight.	Package freight.
Ohio	5 274 313	1 993 278	812 241	366 490
Allegheny	841 923	3 066 969	429 873	746
Monongahela	17 749 150	2 385 099	1 686 122	58 444
Muskingum	21 843	121 526	3 202	5 941
Little Kanawha	320	14 335	7 618	23 678
Kanawha	703 555	163 768	15 340	18 520
Big Sandy	555	0	15 459	13 483
Kentucky	33 655	15 375	139 691	6 651
Green	105 918	3 339	304 085	23 574
Cumberland	95	286 746	73 196	30 669
Tennessee	18 275	962 163	209 210	65 788
Total	24 749 102	9 012 598	3 696 037	613 984-38 071 721
Percentage	65	23.68	9.70	1.62

From Table 5 it will be seen that the Ohio River System is primarily a coal carrier and in the small percentage of other freight resembles some of the principal coal-carrying railroads.

Decrease in Tonnage.—The great decrease in tonnage on the Ohio notwithstanding the improved navigation facilities is due in large measure to a change in the place of marketing the coal mined in the river mines of the Monongahela Valley. In 1901, 3 200 000 tons of coal from such mines were shipped by water to points on the Lower Ohio River. None is shipped to-day either by water or rail.

The recorded decrease is also due in part to the greater care that is now exercised to prevent duplication in the collection of statistics. Some years ago it was the practice on some rivers to arrive at the total tonnage by a summation of the tonnage passing each lock. However, even with such errors eliminated, there has been on some of the rivers in this Basin an actual reduction in the amount of water-borne commerce. A marked exception to this general rule is the Monongahela River, on which the commerce has increased from 7 000 000 tons in 1900, to 12 039 175 tons in 1913, and to an average of 19 558 000 tons during the past six years. The Allegheny River also shows an increase, largely in sand and gravel which made up 75% of its traffic in 1924.

The decline in water-borne commerce has not been limited to the Ohio River Basin, but is more or less general on inland rivers. This subject was brought to the attention of the Society in 1915 in a paper by W. W. Harts, M. Am. Soc. C. E., entitled "'Rivers and Railroads in the United States."* Many reasons for the decline are set forth in that paper and in the voluminous

* *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 919.

discussion thereon. Among the discussors was H. Burgess, M. Am. Soc. C. E., who stated:*

"* * * the more favorable freight rate charged by railroads on any portion of their lines for which there is a competing water route. The result of the lower rates for towns connected by a water route is that 'inland' towns must pay a portion of the rail transportation charge on freight received by river towns. These latter, therefore, receive the superior facilities offered by the rail route over carriage by water at practically what such freight would cost if carried by boat, and these superior facilities are obtained at the expense of the 'inland' towns."

In the same discussion, J. H. Bernhard, Assoc. M. Am. Soc. C. E., after making a series of comparisons between rail rates on lines with and without a competing water route, states:†

"Although rebates are forbidden by State or Federal statutes, railroads grant them quite as much to-day as at any time in the past, the only distinction being that formerly they were granted to individuals surreptitiously, and now they are granted to cities and towns openly; the former became illegal, the latter is legalized. These rebates arise from the unjust practice of railroads in basing rates to river points on water competition."

TABLE 6.—RAILROAD RATES ON COAL, APRIL, 1925.

Place.	KANAWHA DISTRICT.			PITTSBURGH DISTRICT.			OHIO No. 8 DISTRICT.		
	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.
Columbus, Ohio	200	189	9.45	209	189	9.45	145	139	9.58
Dayton, Ohio	288	214	7.43	270	214	7.92	216	183	8.47
Toledo, Ohio	323	239	7.4	260	239	9.19	230	189	8.21
Indianapolis, Ind.	338	252	7.45	388	252	6.49	333	227	6.81
Terre Haute, Ind.	409	287	7.01	460	287	6.26	405	262	6.47
Fort Wayne, Ind.	382	270	7.07	320	270	8.43	296	220	7.43
Columbus, Ind.	317	252	7.98	...	...	....	...	...	....
Bedford, Ind.	360	252	7.00	...	...	....	...	...	....
Mitchell, Ind.	354	252	7.11	...	...	....	...	...	....
Seymour, Ind.	315	252	8.00	...	...	....	...	...	....
Vincennes, Ind.	417	287	6.88	...	...	....	...	...	....
Greensburg, Ind.	291	252	6.88	...	...	....	...	...	....
South Bend, Ind.	465	304	6.53	408	304	7.45	384	254	6.61
Logansport, Ind.	398	304	7.64	398	304	7.64	343	254	7.40
Chicago, Ill.	512	309	6.03	468	309	6.60	444	274	6.17
Peoria, Ill.	549	334	6.08	476	334	7.01	515	309	6.00
Joliet, Ill.	524	309	5.89	...	...	....	...	...	....
Mattoon, Ill.	466	334	7.16	...	...	....	...	...	....
Danville, Ill.	423	309	7.30	...	...	....	...	...	....
Cincinnati, Ohio	228	189	8.29	320	229	7.15	257	214	8.32
Jeffersonville, Ind.	358	198	5.53	450	292	6.48	382	275	7.24
Louisville, Ky.	342	189	5.52	455	292	6.41	378	275	7.27
Evansville, Ind.	497	321	6.46	589	372	6.31	533	347	6.51
East St. Louis, Ill.	564	312	5.53	609	372	6.11	586	347	5.92

To determine to what extent this practice prevails in the Ohio River Basin, existing rail rate tables have been examined and the results which are summarized in Tables 6, 7, and 8, are platted on Figs. 1, 2, and 3. On Fig. 1 is

* *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915).
† *Loc. cit.*, 946.

shown a series of coal rates from bituminous coal fields to points with and without river competition. A curve representing a rate of 4.5 mills per ton-mile plus $1.00 per ton, may be taken as the mean. The short hauls are comparisons between rail rates in the valley of the Monongahela which is canalized and rates in the uncanalized section of the Allegheny. These rates show

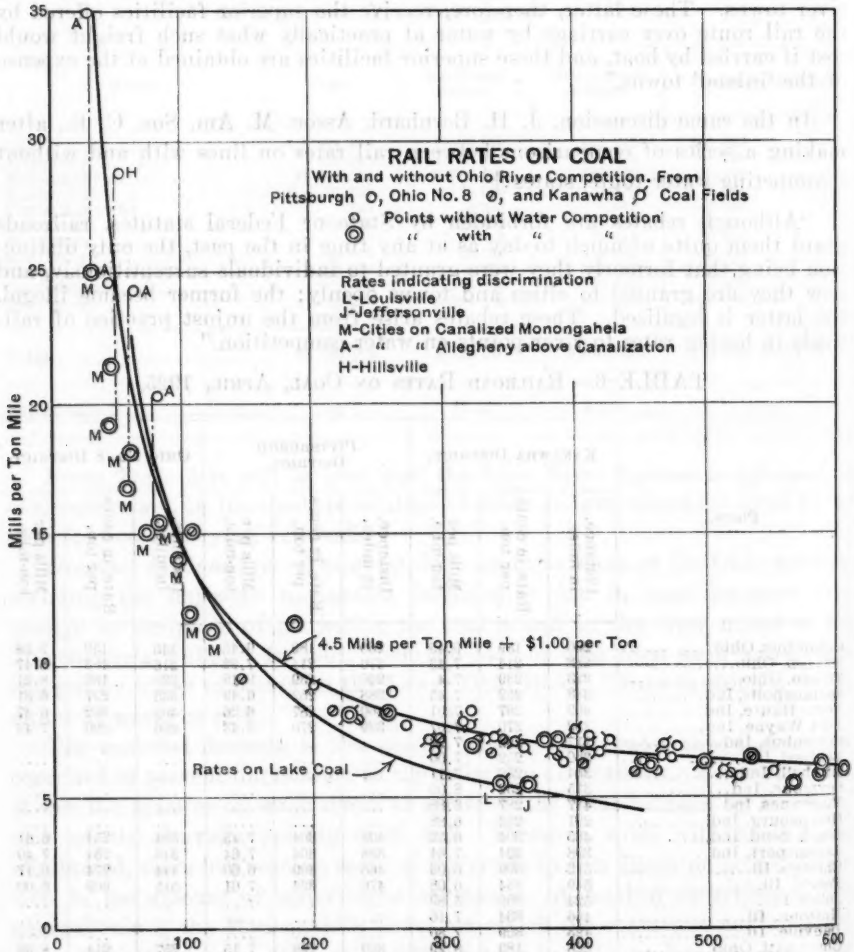

FIG. 1.

marked discrimination in favor of points with water competition, the average difference being 7 mills per ton-mile. In the longer hauls no such discrimination is apparent except on hauls from Pittsburgh to Louisville and to Jeffersonville, Ind. On the same diagram, as a matter of general interest, a line is drawn representing the rates on coal from a number of bituminous coal fields to Lake ports for re-shipment on Lake vessels.

TABLE 7.—Railroad Class Rates, April, 1925.

Cincinnati, Ohio, to:	Distance, in miles.	First-Class Rate.		Fifth-Class Rate.		Sixth-Class Rate.	
		Cents per ton.	Mills per ton-mile.	Cents per ton.	Mills per ton-mile.	Cents per ton.	Mills per ton-mile.
Columbus, Ohio	116	1 170	10.0	410	35.3	330	28.4
Cleveland, Ohio	214	1 550	64.0	540	22.1	430	16.8
Toledo, Ohio	203	1 460	72.0	510	25.1	410	20.2
Chicago, Ill.	285	1 610	56.4	560	19.3	450	15.7
Indianapolis, Ind.	110	1 130	10.3	400	36.3	320	29.1
Terre Haute, Ind.	182	1 420	78.0	500	27.4	400	22.0
Birmingham, Ala.	481	3 020	62.7	1 580	32.8	1 300	27.0
Atlanta, Ga.	488	3 020	61.8	1 580	32.3	1 300	26.6
Pittsburgh, Pa.	311	1 670	53.7	580	18.6	470	15.1
Parkersburg, W. Va.	195	1 450	74.3	510	26.1	410	21.0
Huntington, W. Va.	154	1 310	84.9	460	29.8	370	24.0
Portsmouth, Ohio	107	1 130	105.6	400	37.3	320	29.9
Louisville, Ky.	114	1 130	99.0	400	35.0	320	28.0
Evansville, Ind.	269	1 500	55.8	530	19.7	420	15.6
Cairo, Ill.	374	1 770	47.3	620	16.5	500	13.3
St. Louis, Mo.	339	1 700	50.0	600	17.7	480	14.1
Memphis, Tenn.	494	2 720	55.0	1 410	28.5	1 170	23.6
New Orleans, La.	836	3 740	44.7	1 940	23.2	1 610	19.2
Kansas City, Mo.	618	3 370	54.5	1 210	19.5	1 190	19.2
Nashville, Tenn.	297	103.5	69.7	54	36.3	44.5	29.9
Chattanooga, Tenn.	338	107	63.3	56.5	33.4	47	27.8
Knoxville, Ky.	282	107	75.9	56.5	40.0	47	33.3
Jackson, Miss.	674	171	50.7	89	26.4	73.5	21.8
Meridian, Miss.	634	171	54.0	89	28.1	73.5	23.1
Hattiesburg, Miss.	719	180	50.1	93.5	26.0	77.5	21.5
Montgomery, Ala.	578	152	52.5	80.5	27.8	66.5	23.0
Mobile, Ala.	751	187	49.8	97	25.8	80.5	21.4
Atlanta, Ga.	475	151	63.5	79	33.2	65	27.3
Pittsburgh, Pa., to:							
Cincinnati, Ohio	311	1 670	53.7	580	18.6	470	15.1
Louisville, Ky.	425	1 890	44.5	660	15.5	530	12.4
Evansville, Ind.	580	2 070	35.7	720	12.4	580	10.0
Cairo, Ill.	693	2 510	36.2	880	12.7	700	10.1
Memphis, Tenn.	805	3 240	40.0	1 680	20.8	1 400	17.3
Vicksburg, Miss.	1 026	3 940	37.4	2 050	20.0	1 690	16.4
New Orleans, La.	1 147	4 270	37.1	2 210	19.2	1 840	16.0
St. Louis, Mo.	612	2 450	40.0	860	14.0	690	11.2
Kansas City, Mo.	891	4 120	46.2	1 470	16.5	1 520	17.0
Davenport, Iowa	651	2 450	37.6	860	13.2	690	10.6
Dubuque, Iowa	653	2 450	37.5	860	13.2	690	10.6
St. Paul, Minn.	866	3 360	38.8	1 200	13.8	960	11.1
Cleveland, Ohio	182	1 230	93.1	430	32.5	340	25.7
Detroit, Mich.	316	1 670	52.8	580	18.3	470	14.8
Chicago, Ill.	468	1 950	41.7	680	14.5	550	11.7
Columbus, Ohio	191	1 450	76.0	510	26.7	410	21.4
Indianapolis, Ind.	371	1 770	47.7	620	16.7	500	13.4
Terre Haute, Ind.	443	1 950	44.0	680	15.3	550	12.4
Fort Wayne, Ind.	320	1 670	52.2	580	18.1	470	14.7
Logansport, Ind.	389	1 820	46.7	640	16.4	510	13.1
Peoria, Ill.	561	1 950	34.5	680	12.1	550	9.8
Springfield Ill.	568	2 330	41.0	820	14.4	650	11.4
Birmingham, Ala.	792	4 320	54.5	2 040	25.7	1 680	21.2
Nashville, Tenn.	608	157	50.0	81.5	26.7	67.5	22.2
Chattanooga, Tenn.	649	178.5	55.0	84	26.0	69	21.2
Knoxville, Tenn.	593	178.5	60.0	84	28.3	69	23.3
Jackson, Miss.	985	243	49.3	113	22.9	93.5	19.0
Meridian, Miss.	945	243	51.4	113	23.9	93.5	19.8
Hattiesburg, Miss.	1 080	248	45.2	115.5	22.4	97	18.8
Montgomery, Ala.	889	218	49.0	104	23.4	86	19.3
Mobile, Ala.	1 062	213.5	40.2	110.5	20.8	92	17.3
Atlanta, Ga.	786	219	55.7	108	27.4	88.5	22.5

On Fig. 2 a series of fifth-class rates (bulk products, carload lots) from Pittsburgh and from Cincinnati to points with and without water competition, have been platted. The results show that for points in Central Freight Association territory (that is, the territory bounded on the north by the Great Lakes, on the south by the Ohio River, on the west by the Mississippi River,

and on the east by a line drawn from Buffalo, N. Y., to Pittsburgh), the rates are based on mileage, and no difference is noted between rates to inland points and to points on the Ohio. The rates to many points south of the Ohio, however, are noticeably higher, and some distinction is made between inland points and points on the Mississippi River, such as Memphis, Tenn., Vicksburg, Miss., and New Orleans, La.

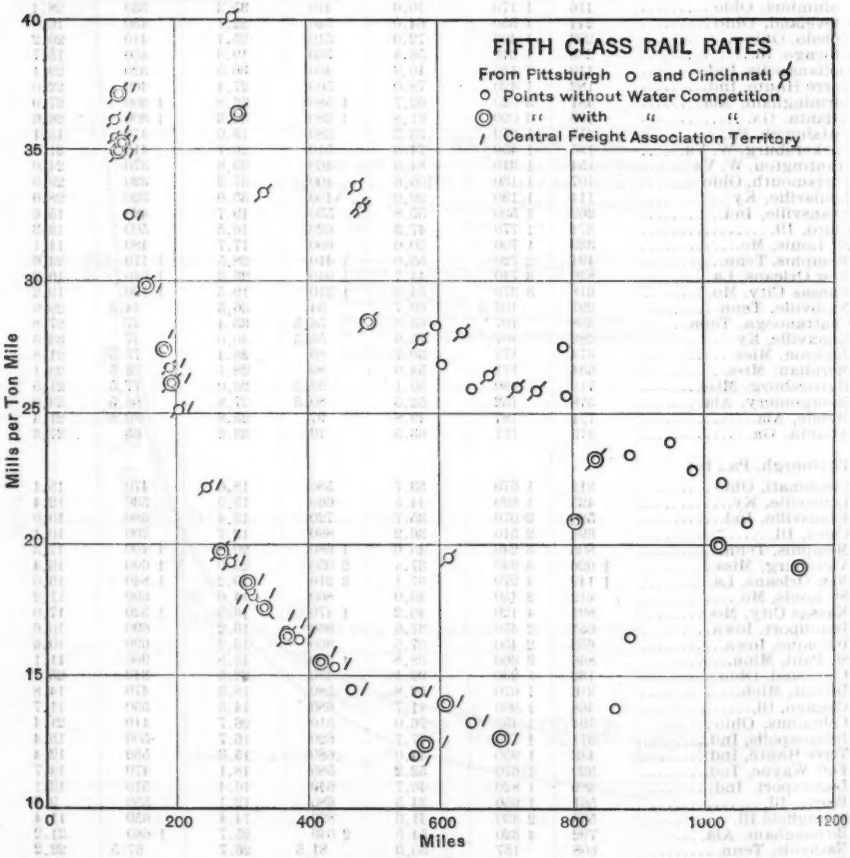

FIG. 2.

On Fig. 3 are platted a series of rail rates on finished steel products from Pittsburgh. In Central Freight Association territory such products move at fifth-class rates. In territory south of the Ohio, they move under a commodity rate which is not only much less than fifth-class rates in that territory, but is practically uniform with fifth-class rates in Central Freight Association territory. The rates indicated on Fig. 3 show no distinction between inland points and points having water competition.

There is a considerable movement of coal from the West Virginia coal fields through Cincinnati to points in Indiana. Some of this coal comes in by rail and is turned over to another railroad for delivery to destination.

Another part comes in by river, is transferred to cars, and delivered to the same railroad for delivery to the same destination, but the rates charged on the coal coming in by rail are lower than the rates charged on the coal coming in by river, other conditions being identical. This differential in favor of the all-rail movement to Indianapolis, Ind., is now 18½ cents per ton and several years ago was considerably higher.

TABLE 8.—Railroad Rates on Finished Steel Products, April, 1925.

Pittsburgh, Pa., to:	Distance, in miles.	Rate.		
		Cents, per hundred weight.	Cents Per ton.	Mills per ton-mile.
Cincinnati, Ohio	311	29	580	18.6
Louisville, Ky	425	33	660	15.5
Evansville, Ind	580	36	720	12.4
Cairo, Ill	693	44	880	12.7
Memphis, Tenn	805	56	1 120	13.9
Vicksburg, Miss	1 026	64	1 280	12.4
New Orleans, La	1 147	67	1 340	11.7
St. Louis, Mo	612	43	860	14.0
Kansas City, Mo	891	73.5	1 470	16.5
Davenport, Iowa	651	43	860	13.2
Dubuque, Iowa	653	43	860	13.2
St. Paul, Minn	866	60	1 200	13.8
Cleveland, Ohio	132	19	380	28.8
Detroit, Mich	316	29	580	18.3
Chicago, Ill	468	34	680	14.5
Columbus, Ohio	191	25.5	510	26.7
Indianapolis, Ind	371	31	620	16.7
Terre Haute, Ind	443	34	680	15.3
Fort Wayne, Ind	320	29	580	18.1
Logansport, Ind	389	32	640	16.4
Peoria, Ill	561	34	680	12.1
Springfield, Ill	568	41	820	14.4
Birmingham, Ala	792	58	1 160	14.6
Nashville, Tenn	608	50	1 000	16.4
Chattanooga, Tenn	649	50	1 000	15.4
Knoxville, Tenn	593	50	1 000	16.8
Jackson, Miss	985	64	1 280	13.0
Meridian, Miss	945	62	1 240	13.1
Hattiesburg, Miss	1 030	64	1 280	12.4
Montgomery, Ala	889	62	1 240	13.9
Mobile, Ala	1 062	67	1 340	12.5
Atlanta, Ga	786	58	1 160	14.7

With the exceptions noted, the results of the investigation seem to justify the general statement that in the movement of coal and structural steel in the Ohio River Basin, the existing rail-rate structure does not discriminate between points having water competition and points that are without water competition.

Cost of Transportation by Water, Ohio River System.—In determining the economic advisability of creating an inland waterway or in measuring its usefulness as a part of the country's general transportation system, one of the most important factors and one concerning which available information is limited is the cost of water haulage, that is, the cost of transportation by water exclusive of fixed charges. This cost is subject to the combined influence of many factors, among which may be listed:

(*a*) The physical characteristics of the waterway, of which in canalized rivers the more important are the size of the locks and the depth and width of the intervening pools.

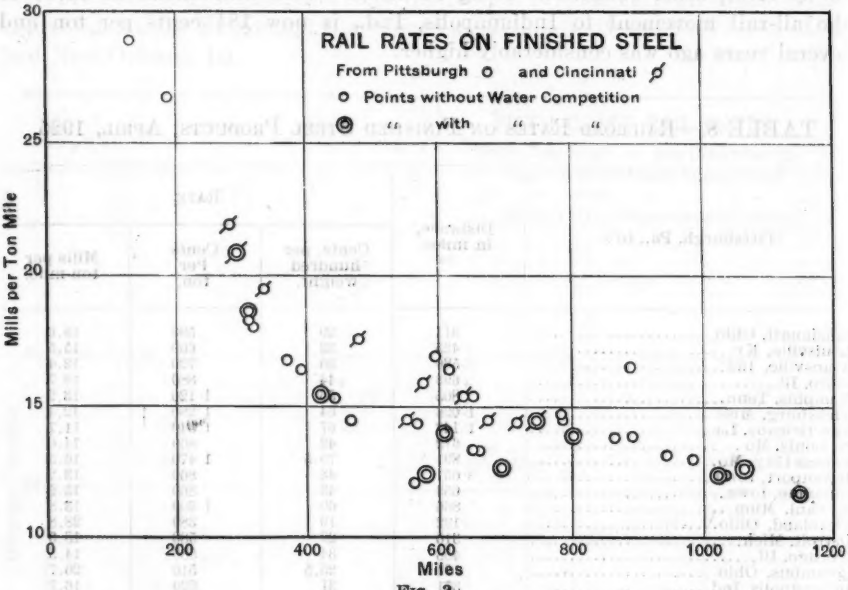

FIG. 3.

(*b*) Types of transportation service, which may be grouped as private industrial, contract industrial, and common carrier.

(*c*) The kind of freight: In the comparisons to follow only bulk freight will be considered and in some cases only coal.

(*d*) Type of floating plant: As more than 98% of the tonnage on the Ohio and its tributaries is carried on barges, open or decked, no consideration will be given to a movement of freight on packet-boats or other self-propelled carriers.

(*e*) Annual volume of tonnage: The volume of tonnage handled annually by a given operating agency affects the number of barges required per towboat, and the plant performance. On the Monongahela River, with its large volume of tonnage, a standard tow consists of six barges, each approximately 175 ft. long and 26 ft. wide, with a maximum capacity of 1 000 tons. These barges are lashed together, two wide, and pushed by a stern-wheel steamer, the overall dimensions of which are somewhat less than a standard barge. For passage through a standard Monongahela River lock such a tow must be broken into two parts. On the Ohio a fleet of eight such barges with towboat can pass through the locks. Tows with twelve to fourteen barges are not infrequent on the Ohio between the mouth of the Kanawha River and Cincinnati, and formerly even larger tows were not uncommon.

(*f*) Back haul: Very little freight is available as back haul that can be carried on barges used for the transportation of coal, sand, and gravel. Where

back-haul traffic is available, the costs of transportation are from 60 to 65% of those without back haul.

(*g*) Length of haul: This factor influences the investment per ton-mile and the plant performance.

(*h*) Co-ordination of terminal and transportation services: This factor gives private industrial operation an advantage over contract industrial or common carrier and results in lower costs.

In a discussion of the paper presented by William M. Black, M. Am. Soc. C. E., entitled "Waterways and Railway Equivalents",* C. I. Grimm, Assoc. M. Am. Soc. C. E., analyzed† waterway haulage costs on rivers similar to the Ohio, and reached the conclusion that 2.5 mills per ton-mile was a fair measure of the cost under the following conditions: 250-mile haul, no back haul, 6 000-ton cargoes in 1 000-ton barges, wide river, large tonnage, and steady movement.

The principal users of the Monongahela River have kept comprehensive cost records for a number of years. These records were furnished in confidence to the office of which the speaker is in charge and, together with the analysis made by Mr. Grimm, have been made the basis of the water haulage costs shown on Fig. 4. These costs are based primarily on tonnage moving with the current. Other conditions are shown on the diagram itself.

Rail vs. Water Rates.—Fig. 4 also shows existing rail rates on coal on roads coming into direct competition with the Ohio River or its coal-carrying tributaries. Ton-mile rates by rail and water are, however, not directly comparable, one important variation being the difference in the length of the haul.

Fig. 5 shows water and rail distances from Pittsburgh to points on the Ohio and the Mississippi Rivers. For a rail haul of 230 miles the water distance is approximately 50% greater than the rail distance, and for convenience in comparison with rail distance, a line representing this ratio, has also been shown. On the Monongahela River the competing railroads parallel the river, and differences in lengths of haul are insignificant.

Another point that must be taken into consideration in any economic comparison is the difference in terminal costs. Experience on the Monongahela River indicates that it is somewhat cheaper to load into cars than into barges. On the Ohio the difference is greater, due to the greater range between normal and high-water stages. Coal cars can be unloaded from a trestle or elevated and tipped bodily, while coal barges must be unloaded by a grab-bucket. With modern unloading plants of large capacity this difference amounts to less than 5 cents per ton.

Every transfer of bituminous coal, whether loading or unloading, involves some loss due to breakage and, as this breakage is generally proportional to the drop, loading into barges has been considered as involving greater loss than loading into cars. To minimize this loss, three modern plants in Huntington, W. Va., for transferring coal from rail to barge are in the form of inclined belt conveyors. For joint hauls involving two railroads, cars can

* *Proceedings*, Am. Soc. C. E., August, 1924, Papers and Discussions, p. 837.
† *Loc. cit.*, February, 1925, Papers and Discussions, p. 320.

be switched from one to the other at small cost, whereas in joint hauls—part river and part rail—both a transfer charge of about 10 cents per ton and a breakage loss must be absorbed.

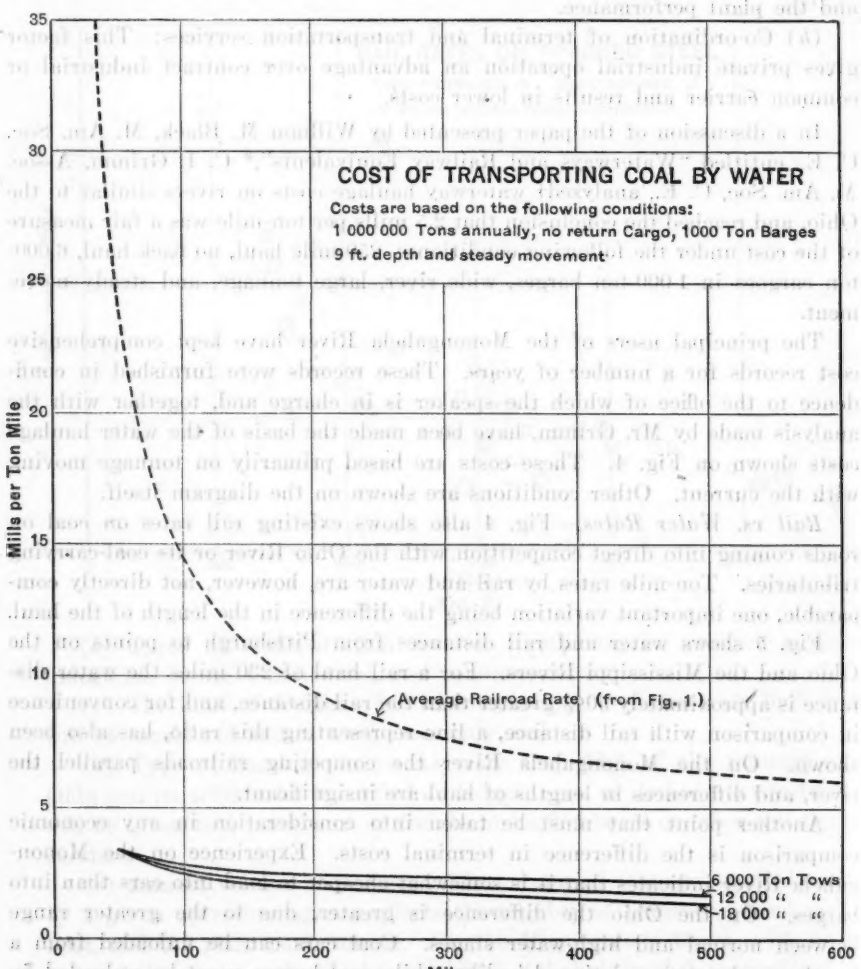

FIG. 4.

Economic Comparisons.—If the points of origin and destination and the ton-mileage of each commodity hauled on these rivers are known, as well as the water haulage costs, the direct benefits or savings resulting from their use can be readily ascertained by comparison with existing rail rates and their economic justification determined by comparing the savings or benefits with the fixed charges. Although neither the traffic data nor the water-haulage cost data are sufficiently comprehensive for a complete comparison, approximate results can be secured by limiting the comparison to coal and assuming

that the ton-mile savings on that commodity represent the average savings on the total ton-milage.

The average cost of moving coal on the Monongahela River is quite well determined and is 3 mills per ton-mile and the rail rate on coal on parallel roads for the same average haul—42 miles—is 19 mills per ton-mile, the rail and water distances being practically equal in this case. This difference applied to the total ton-milage—919 000 000—reported for 1924, shows a gross annual saving of $14 700 000, or allowing for difference in terminal costs, a net annual saving of $12 700 000, in comparison with fixed charges of $1 729 980.

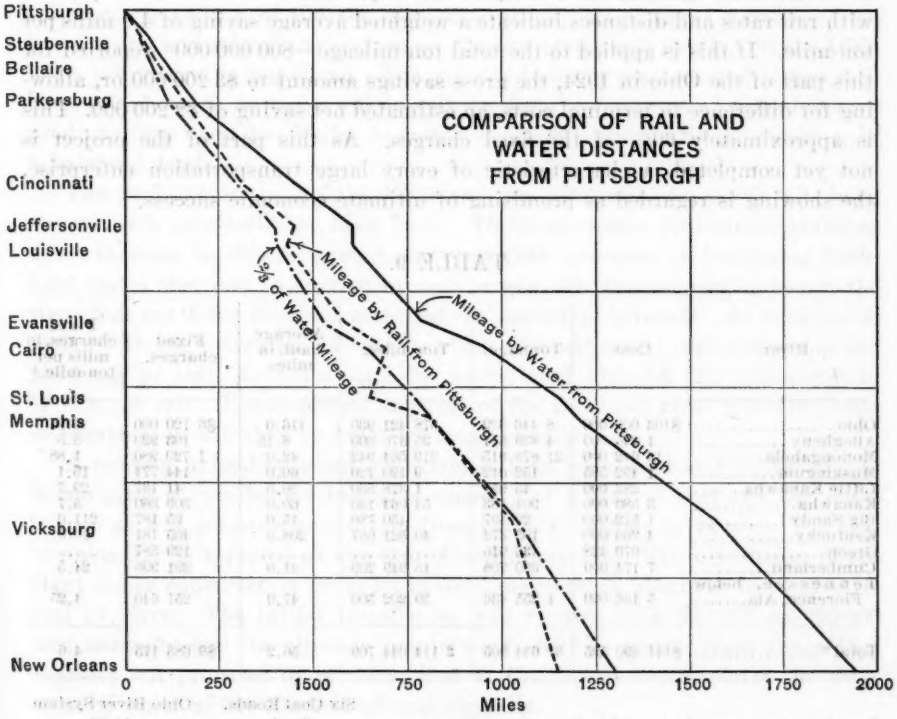

FIG. 5.

A similar comparison on the Ohio is limited to the reach between Pittsburgh and Cincinnati, 483 miles in length. There are only three uncompleted dams above Cincinnati and as they will be completed in 1925, the cost of that part of the project and the cost of operation and maintenance can be closely approximated. With a first cost of $54 500 000, interest at 4%, and operation and maintenance cost of $1 500 000, the fixed charges will be $3 680 000.

The coal moving on the Upper Ohio may be divided into three groups:

(*a*) That coming out of the Monongahela River and moving to points in the industrial district between Pittsburgh and Wheeling;

(*b*) That coming out of the Kanawha; and

(*c*) That coming into Huntington, W. Va., by rail and being transferred to barge.

Part of Movements (*b*) and (*c*) goes to the industrial district at Ashland and Ironton, Ohio, but the major part goes to Cincinnati. Some of the latter coal is re-shipped by rail, involving a rail-river-rail movement, the coal being hauled from Guyan Valley by rail, 70 miles, transferred to river barges, hauled by water to Cincinnati, 162 miles, and re-shipped by rail to points in Indiana.

The computation of direct benefits on the Upper Ohio is not as simple as on the Monongahela, but a study of the major movements and a comparison with rail rates and distances indicate a weighted average saving of 4.0 mills per ton-mile. If this is applied to the total ton-mileage—800 000 000—reported for this part of the Ohio in 1924, the gross savings amount to $3 200 000 or, allowing for difference in terminal costs, an estimated net saving of $2 200 000. This is approximately 60% of the fixed charges. As this part of the project is not yet completed, a characteristic of every large transportation enterprise, the showing is regarded as promising of ultimate economic success.

TABLE 9.

River.	Cost.	Tonnage.	Ton-miles.	Average haul, in miles.	Fixed charges.	Fixed charges, in mills per ton-mile.†
Ohio	$103 000 000	8 446 322	978 421 930	116.0	$6 120 000	6.25
Allegheny	1 446 000	4 339 511	35 370 300	8.15	193 920	5.5
Monongahela	13 612 000	21 878 815	919 564 942	42.0	1 729 980	1.88
Muskingum	1 452 595	152 012	9 120 720	60.0	144 774	16.1
Little Kanawha	282 000	45 951	1 378 530	30.0	41 137	29.7
Kanawha	3 580 000	901 183	54 064 140	60.0	309 360	5.7
Big Sandy	1 512 000	29 497	450 780	15.0	95 187	211.0
Kentucky	4 266 000	195 372	40 521 667	208.0	405 181	10.0
Green	670 428	436 916	.....	.....	129 587	.....
Cumberland	7 174 000	390 706	15 949 200	41.0	391 996	24.5
Tennessee, below Florence, Ala.	5 166 000	1 255 436	59 202 500	47.0	251 640	4.25
Total*	$141 490 595	37 634 805	2 114 044 709	56.2	$9 683 175	4.6

	Six Coal Roads.	Ohio River System.
Investment per annual ton (not including equipment)	$3.50	$3.75
" " " ton-mile	0.016	0.0675
Fixed charges per ton-mile	0.002	0.0046

* Not including Green River.

† The fixed charge per ton-mile for the entire system is 4.6 mills, 2.3 times the fixed charge per ton-mile on the principal coal-carrying railroads in this region. If upon completion of the Ohio canalization project the commerce is doubled, as seems probable, the fixed charge per ton-mile will be quite comparable with that on coal-carrying railroads.

Similar comparisons made on other tributaries show some as yielding benefits comparable with the fixed charges, others have not yet reached the stage of economic justification, while still others have apparently passed beyond it, as did the old canals constructed 100 years ago. Certain data concerning the Ohio River System as a whole for 1924 are shown in Table 9.

On the lower half of the Ohio, the tonnage for 1924 amounted to 1 400 000, of which, 420 000 tons was a seasonal movement of coal from the West Kentucky fields to Paducah, Ky., Cairo, and points on the Mississippi River. With the completion of the canalization project this tonnage bids fair to increase greatly. There is now a seasonal movement of steel products from Pittsburgh to Memphis and Baton Rouge, La., where two of the large steel corporations have established distributing points for supplying the Southwest; 66 000 tons were shipped in 1924 and with year-round navigation the tonnage will increase.

On the Mississippi River, in 1923, there was an up-stream tonnage (Memphis to Cairo), of 400 000, consisting principally of sugar, molasses, logs, bauxite ore, and miscellaneous items. Similar traffic will develop on the Ohio when the project is completed. For some of this tonnage, covered barges will be required which, in turn, will be suitable for the shipment of manufactured products from the Ohio Valley to points on the Mississippi River, resulting in a balanced haul.

The coal movement on the Ohio and its tributaries is nearly all down stream with practically no back haul. There are many indications pointing to an increase in this movement, but very little prospect of developing back-haul traffic that can be carried in coal barges. In this respect, however, the river does not differ from the principal coal-carrying railroads. As evidence of increase in the movement of coal by river, two additional coal companies are planning to ship, to Cincinnati by water, coal brought into Huntington, W. Va., by rail. The combined capacity of the three rail-river transfer plants at Huntington will then be 4 000 000 tons annually.

A modern unloading plant has recently been completed at Cincinnati. It is equipped with a bucket having a capacity of 5 tons of coal and the speed is such as will permit a round trip from barge to hopper in 40 sec. The cost of unloading is reported as less than 10 cents per ton. The new steam power plant under construction 20 miles below Cincinnati is planning to receive its coal by river. The initial installation will require from 250 000 to 300 000 tons annually and its ultimate requirements will be nearly three times that quantity. A proposed by-product plant at Cincinnati contemplates the shipment by water of 500 000 tons of coal annually.

The steel-making industries on the Ohio are responsible for the largest part of the present coal movement, 2.6 tons of coal being required directly or indirectly in the manufacture of each ton of steel, and if these districts continue to grow or other similar districts are established, the Ohio may eventually be as essential to their success as the Monongahela River is to the steel industries in that valley.

Recently, in connection with its Parkersburg, W. Va., refinery, the Standard Oil Company established a water delivery service for its refined products. With a fleet of six steel barges, with a capacity of 100 000 gal. each, deliveries are being made to points on the Ohio between Wheeling and Huntington and to points on the Kanawha River. Another recent development is the

movement of 100 000 tons annually of crude oil from points on the Kentucky River to the Ohio, thence to Louisville.

Many of those interested in the completion of the Ohio River project look forward to such reduction in transportation costs as to make Baton Rouge and New Orleans the exporting points for a great part of the Ohio Valley instead of points on the Atlantic. Others foresee the time when iron ore will be brought by sea to Baton Rouge and barged up the Mississippi and Ohio Rivers to meet the coal; but even if some of these forecasts are not realized, it seems probable that the canalization of the Ohio will eventually yield benefits in excess of the fixed charges and thus justify the investment.

By some it is held that inland waterway improvements are justified even if the benefits are less than the carrying charges, but in its report to Congress, a number of years ago, the National Waterways Commission stated that:

"The cost of facilities for carrying freight, whether borne by the Federal Government or by private capital, is a burden upon the resources of the country. The fundamental criterion should be whether a railway or a waterway, when constructed or improved, will be a profitable investment of capital."

The same view was expressed recently by General Black in his paper on "Waterway and Railway Equivalents", previously mentioned, as follows:

"New waterway lines of transportation should not be established unless an analysis of the existing railway, highway, and waterway lines and of the need for transportation shows that the proposed new line is required and that if established it will produce an annual saving in the cost of transportation greater than the interest on construction plus maintenance and operating costs."

The writer would phrase a similar thought by saying that the people of the United States are entitled to have the products of the farm, mine, and factory move from origin to destination by the cheapest means, all elements of cost considered. Every inland waterway project should be gauged on that basis. Projects that are economically sound should be prosecuted vigorously and their use encouraged by wise regulation of rates and by the establishment of joint rail and water rates so as to make them an integral part of the country's transportation system.

THE RELATION OF THE OHIO-MISSISSIPPI WATERWAYS TO PRODUCTION AND INDUSTRIAL DEVELOPMENT

By H. B. Luther,* M. Am. Soc. C. E.

The proper utilization of the Ohio-Mississippi Waterways for transportation is rapidly becoming essential to the safeguarding of National prosperity and to the proper economic and industrial development of the United States. This is considered to be a fair statement because two distinct types of studies have indicated that these waterways afford a means of solving important industrial problems. This particular paper is confined to a description of these two fundamental studies and the relation of the Ohio-Mississippi route to them. The first was a war-time study of production which is equally applicable to the general problem of production, and it stresses a serious situation which is worthy of frequent re-statement from all angles. The second study is concerned with the industrial development of the Cincinnati District.

Prosperity is linked with production. Periods of prosperity make demands on the entire capacity of the Nation for production, and in time of war the country's ultimate military effort is limited by production. There are seven limiting factors of production, namely: (1) Demand; (2) fluid capital; (3) transportation; (4) labor; (5) raw material; (6) equipment; and (7) power.

It is impossible to place these factors in the order of their importance, because at different periods any one of the various items may become the limiting factor. At present (1925), demand is the limiting factor in production. The country is so equipped with labor, raw materials, and sources of power in its coal supply and water-power possibilities, that the proper handling and utilization of these commodities should make them meet any demands that can be anticipated. At critical periods, however, transportation is the real limiting factor of production. The great panic of 1907 was caused fundamentally by the inadequacy of transportation facilities, and the transportation facilities again failed to meet the demand after the entrance of the United States into the World War.

That Division under the U. S. Chief of Ordnance, which handled economic problems, emergency production, and labor matters, made a survey of transportation conditions in the fall of 1917. The flow of material was primarily eastward. Congestion appeared very early. Outline maps of the United States, such as are used by the U. S. Weather Bureau, were prepared with each railroad yard represented by a red dot. The congestion in each yard

* Prof. of Civ. Eng., Univ. of Cincinnati, Cincinnati, Ohio.

was indicated by an arrow pointing backward from the yard, westward. This arrow lengthened week by week and the rate at which this length increased was the basis for the prediction of the date of complete congestion. In December, 1917, it was clear that this would occur by February 1, 1918. Additional factors—in particular, severe weather conditions—hastened the tie-up and by January 17, 1918, the Coal Commission issued its orders closing down all factories for one week and for one day per week thereafter, until the congestion was cleared. It was necessary to suggest many changes in orders for war material which various Departments had placed in plants admirably adapted by equipment and personnel, but situated where material could not be shipped in or out with needed regularity.

Herman Schneider, Assoc. M. Am. Soc. C. E., who was closely concerned with those studies, gives a simple illustration of the transportation situation in one of his professional papers. The transportation facilities correspond to the aisles of a factory along which the raw materials are carried to the machines and the finished products are carried away from them. There is an efficient width of aisle which will avoid congestion and consequent tie-up of various machines. Further, the proper layout of the equipment must be considered in its relation to a proper sequence of functions and a minimum transport of material from the entrance of the plant to its issue as finished product. Individual plant layout is a matter of careful analysis. The industrial layout of the entire country should be viewed in much the same manner, treating the country as one vast factory. At present, the aisles are not wide enough nor is the machinery in all cases properly placed.

In 1904, there was $1 in railroad property for each $9 of other property; in 1912, the ratio was 1 to 12, and, in 1920, there was nearly $25 worth of other property for each $1 of railroad plant. Although the basis of valuation has changed somewhat in that period, the comparison of these ratios is an important index. In case of a war, the nation's efforts would still be limited by transportation. It is stated that during the period of congestion on the Eastern seaboard, under World War conditions, it was easier to ship goods from Pittsburgh, Pa., to New York, N. Y., *via* New Orleans, La., than from Pittsburgh to New York direct by rail. On the completion of the work on these waters, a similar tie-up into the North Atlantic ports would make exporting by way of New Orleans feasible, that is, it would be easier to ship from Cincinnati, Ohio, to New Orleans by river and then by boat to France than to ship from Cincinnati to New York and export from there. The increased utilization of these waterways is necessary for general business demands and as a measure of National security.

Again the matter may be viewed from a study of proper industrial development. Such development for any particular locality should be based on sound searching studies of all the economic principles involved and on a proper consideration of the industrial relation of that particular section to the country at large. In the past, industries have been encouraged for the purpose of building up communities, without due care in the selection of the proper type of industry. There are exceptions, of course, primarily in the location near the source of supply of those industries using bulky raw materials.

In this problem of the selection of proper industries, the Ohio-Mississippi Waterway is an important factor for those States bordering on it. Investigations of the proper industrial developments of the Cincinnati District are under way. The work has not yet reached the stage where direct recommendations can be made, but a discussion of the fundamental principles involved, with stress on the question of transportation, together with an illustration of the specific type of problem which arises, will indicate the importance attaching to the waterways.

The question of the proper industry for a site involves many factors. First, the natural resources of the surrounding country should be studied; the demands of the community must be analyzed; the type of power and labor available must be known; and the contacts with neighboring industrial centers considered.

For example, assume that a plant to manufacture a given commodity is to be located in Cincinnati and that a request is made for capital. In the investigation of the wisdom of such investment it is necessary, in order to determine the transportation charges, to determine the sources of supply of its raw materials and to learn the dependency to be placed on a continual flow of this supply. Then, with the cost of the finished product known, there may be determined the distance this product can be sent in competition with similar factories which may be located in Chicago, Ill., St. Louis, Mo., and Pittsburgh. The demand for this product in the territory it may economically supply must be studied and the output of existing local plants determined. Obviously, if the demand in the local marketing area is already 90% filled by existing agencies, a 25% increase in production capacity would mean failure of some of the companies or severe financial stress for all. This leads toward the question of decentralization for certain types of industries.

In order to have a basis for solving these industrial problems, two surveys have been started in Cincinnati. The first is determining the natural resources available to the Cincinnati District. The area to be studied varies with the type of material, in some instances a radius of 50 to 100 miles is taken, in others, one of 500 to 1 000 miles, depending on the bulk of the commodity and the means of transportation. This work is being carried on by the University of Cincinnati, with aid from the Commercial Club. At the same time, a study of the existing manufactures in this District, the demands in raw materials at present made by these plants, and the output from this District, is being carried on by the Technical Advisory Corporation. This work is supported financially by the Chamber of Commerce.

To illustrate how Ohio River transportation has entered this study, the following illustration serves excellently. Among the studies being made in connection with these investigations has been a review of the uses of salt and the sources from which it is now obtained for local purposes. At present, salt is brought chiefly from Northern Ohio and Michigan. Rock salt is mined somewhat more cheaply in New York State, but transportation charges make the Northern Ohio and Michigan sources more available. However, rock salt can be obtained in large quantities in Louisiana, a source which bids fair to

become of great importance in supplying the demands of the country. It may be brought from this supply in barges on the Ohio and Mississippi Rivers and delivered to factories established on the banks of the Ohio at favorable rates in competition with the present sources. There it can be crushed and screened and the demands for various sizes of rock salt for all local needs in the chemical industry supplied. The excess of all sizes can then be used at these new factories on the river to manufacture certain chemicals. Alkali (sodium hydroxide), suggested as the lime needed in conjunction with it, is available in great plenty in this region, and the electrolytic manufacture of sodium hydroxide could use any excess electric power if this proves feasible. Salt cake for the glass industry has also been suggested.

Also, salt cake (Na_2SO_4) demands sulfur, and this demand carried the investigation to a study of the source of this raw material. The sulfur used at present comes chiefly either from the Eastern seaboard, from places such as Perth Amboy, N. J., where it is obtained from pyrites imported in bulk, or from pyrites found in Tennessee. Sulfur may be obtained practically pure from wells in Louisiana and Texas and transported on the inland waterway. With cheap sulfur for the manufacture of sulfuric acid, an ample salt supply, and adequate power, the materials are assembled for the maintenance of a heavy chemical industry, for the manufacturing of such products as the salt cake, already mentioned, soda ash, caustic soda, and hydro-chloric acid, all of which are products that find places in the industrial demands of Cincinnati.

This is the type of problem now confronting those engaged in the investigations of accessible materials and their utilization. The feasibility of this particular manufacture is still to be determined. However, recent studies seem to indicate that the Ohio-Mississippi route will place rock salt and sulfur among those raw materials so situated geographically as to fall among the natural resources which determine the proper industry for this locality.

There is a local demand for pig iron, and the question has been asked, should blast furnaces be located in the more immediate vicinity of Cincinnati? The transportation factor involved is that of barging coal from the Pittsburgh region in West Virginia—a water haul—and the rail shipment of the ore from a Lake port, such as Lorain, Ohio, a haul comparable to the haul from the Lakes to Pittsburgh. Local factories that use by-product coke and slag cements deserve consideration in this connection, as well as the practicability of smelting at Cincinnati, a mixture of Birmingham iron, Bessemer and Lake Superior ores. The use of sulfur and salt is stressed, however, to indicate the need of an investigation of the natural resources of the Ohio-Mississippi Basin as a means to intelligent industrial expansion, along proper lines, of those industrial centers adjacent to the waterway.

COMPARISON OF RAIL AND RIVER TRANSPORTATION COSTS IN RELATION TO THE OHIO RIVER AND ITS TRIBUTARIES

By Frank H. Alfred,* M. Am. Soc. C. E.

The relation of the Ohio River and its tributaries to transportation in the United States may mean either of two things: First, how the one compares with the other as to amount, importance, expedition of service, economy, or some other characteristic; and, second, how the two are connected with or related to each other—the nature and extent of their interdependence and interaction. The first is largely a matter of statistics. The salient facts in regard to transportation on the Ohio River† are contained in Tables 10 and 11.

TABLE 10.—Freight Traffic, 1923.

Commodities.	Net tons.	Percentage of total.	Value.	Percentage of total.
Coal and coke	6 041 708	73.0	$27 466 401	25.0
Sand, gravel, and stone	1 372 404	16.6	1 542 881	1.4
Metals and metal products	389 792	4.7	36 714 894	33.4
All other commodities	476 617	5.7	44 298 077	40.2
Total	8 280 520	100.0	110 022 253	100.0

The figures given in Tables 10 and 11 include freight handled in car ferries. Since 1918 this item has been kept separately, and figures exclusive of car-ferry traffic are, as follows:

Year.	Tons.	Value.
1919	5 004 378	$88 912 336
1920	9 382 464	223 433 492
1921	7 307 880	93 274 814
1922	6 291 828	144 796 250
1923	8 280 520	110 022 253

Characteristics.—The characteristics of the Ohio River are, as follows:

Length, in miles		968.5
Drainage area, in square miles		203 900
Discharge, in cubic feet per second (estimated):		
	Low water.	High water.
At head, Pittsburgh, Pa	1 600	440 000
At mouth, Cairo, Ill	27 000	1 500 000

* Pres. and Gen. Mgr., Pere Marquette Ry., Detroit, Mich.

† Annual Report, Chf. of Engrs., U. S. Army, 1924.

Average fall per mile at low-water stage:

	Miles.	Inches.
Pittsburgh, Pa., to Wheeling, W. Va.	90	11½
Wheeling, to Cincinnati, Ohio.....	378½	5½
Cincinnati, to Cairo, Ill.....	500	4

Construction of Locks and Dams.—The existing project for improvement was adopted by the River and Harbor Act of June 25, 1910, and contemplated a 9-ft. channel from Pittsburgh to Cairo. It includes fifty-two dams, with a lock, 110 ft. by 600 ft., at each dam. Of these, thirty between Pittsburgh and Portsmouth, Ohio, are in operation, being all that are contemplated between those points, a distance of 358 miles. Between Portsmouth and Uniontown, Ky. (480 miles), seven dams are in operation, nine are under construction, and one is not yet begun. Between Uniontown and Cairo (130 miles), the five dams contemplated are not yet begun. This makes a total of thirty-seven in operation, nine under construction, and six not begun, on June 30, 1924.

TABLE 11.—FREIGHT TRAFFIC, 1892-1923.

Year.	Tons.	Value.
1892	6 901 186	.....
1893	7 371 804	.....
1894	7 795 501	.....
1895	7 963 478	.....
1896	9 914 435	.....
1897	11 265 638	.....
1898	6 756 627	.....
1899	13 529 742	.....
1900	14 054 322	.....
1901	10 064 978	.....
1902	12 202 017	.....
1903	12 499 842	.....
1904	10 142 551	.....
1905	13 163 656	.....
1906	11 427 784	$79 994 488
1907	11 306 544	79 145 808
1908	8 498 754	59 491 278
1909	8 676 701	60 736 907
1910	11 112 216	358 809 889
1911	12 046 294	82 074 878
1912	8 618 369	71 064 229
1913	9 814 123	77 026 901
1914	9 530 309	93 294 479
1915	9 273 184	188 510 914
1916	7 917 112	165 130 487
1917	6 149 213	309 272 755
1918	6 662 941	145 654 917
1919	5 469 424	186 710 382
1920	9 869 325	368 607 567
1921	8 037 788	464 576 262
1922	7 039 138	968 327 994
1923	9 245 647	372 534 148

In 1910, prior to the adoption of the present project, $17 657 273 had been expended, with $58 974 278 since that time, making a total of $76 631 551. The estimated cost of completing the project is $34 084 625, which would make the total cost $110 716 177. The cost of maintenance and operation in the year ending June 30, 1924, was $1 420 574.

The classification of freight in 1923 by principal commodity groups and by up-stream and down-stream movement, with the ton-miles and average haul in each case, is given in Table 12.

TABLE 12.—FREIGHT CLASSIFICATION, 1923.

UP STREAM.

Commodities.	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	343 225	20.6	13 563 726	40
Sand, gravel and stone	972 645	58.2	10 946 066	11
Metal and metal products	167 055	10.0	22 353 055	134
All other commodities	187 361	11.2	75 233 212	401
Total	1 670 286	100.0	122 096 059	73

DOWN STREAM

	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	5 698 483	86.2	489 752 210	86
Sand, gravel and stone	399 759	6.0	6 328 085	16
Metals and metal products	223 737	3.4	101 619 654	456
All other commodities	289 256	4.4	29 324 478	101
Total	6 610 235	100.0	627 024 427	95

TOTAL.

	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	6 041 708	73.0	503 315 936	83
Sand, gravel and stone	1 372 404	16.6	17 274 151	13
Metals and metal products	389 792	4.7	123 972 709	318
All other commodities	476 617	5.7	104 557 690	219
Total	8 280 521	100.0	749 120 486	90

No attempt to compare Ohio River traffic with all classes of transportation in the United States will be made, as that would include not only other water-borne traffic, but also that carried on the highways. The comparison will be confined to the freight traffic carried by the railways. This is shown in the following statement (Table 13) for 1923, the last year in which statistics as to river traffic are available.

In this comparison (Table 13) it should be noted that the cost of railroads includes rolling stock, which amounts to about one-third of the total cost, whereas the cost for the river does not include boats and barges, nor does it include anything to correspond to shops, fuel, and water stations, freight and passenger stations, or terminal facilities in general, which form so large a part of the cost of a railroad. It is really equivalent to the cost of the roadway only, which, in the case of the Pere Marquette Railway, amounts to only about one-third of the total.

TABLE 13.

	Railroads in United States.	Ohio River.	Percentage of railroad.
Length, in miles	250 183	968.5	0.39
Tons of freight	2 383 000 000	8 280 520	0.35
Tons of freight per mile of length	9 065	8 550	94.3
Ton-miles	416 211 000 000	749 120 486	0.18
Ton-miles per mile of length	1 663 228	773 485	46.5
Cost, including equipment	$22 681 000 000	.....	.....
Cost per mile including equipment	$ 90 658	.....	.....
Estimated cost of improvement	.....	$110 716 177	.....
Estimated cost per mile of improvement	.....	$ 114 318	.....

In 1923, the average railway revenue per ton-mile was 1.116 cents. On the Pere Marquette Railway the average revenue per ton-mile was 1.118 cents, which was disbursed, as follows:

	Cents.
Maintenance of way and structures	0.141
Maintenance of equipment	0.233
Transportation	0.390
Traffic, general and miscellaneous	0.063
Taxes	0.044
Rentals	0.071
Interest	0.053
Surplus	0.123
Total	1.118

This distribution is on the assumption that the ratio of expense to revenue is the same for freight as for all traffic.

Data are not available for arriving with accuracy at corresponding figures for Ohio River traffic. However, the uncertain factors are not important, so that a total may be reached, which is fairly comparable with that for railroads.

The improvement project will probably be completed within the next three or four years at a cost now estimated at $110 000 000, on which interest at 4% would be $4 400 000. The maintenance in 1923, with thirty-seven dams in operation, cost $1 420 574, so it may be assumed that the cost of maintenance of the completed project will average $2 000 000 per year.

Depreciation on the railroads is included in their maintenance cost, but not in the figures given for the river. In a paper on "Traffic on the Ohio River"*, Mr. Charles Whiting Baker states that: "The first dams built on the Ohio River already require reconstruction. A depreciation charge of 1½% allows at least as long a life for these works as can safely be assumed". This rate gives a yearly depreciation charge of $1 650 000.

The Government, by making this expenditure, will have withdrawn $110 000 000 from the taxable wealth of the nation, on which otherwise it would be receiving taxes. A fair comparison with railroad costs, therefore, should

* *Engineering News-Record*, January 15, 1920.

include taxes on this amount. In 1923 the railroads paid taxes of $332 000 000 on an investment of $22 681 000 000, or at the rate of 1.46%, which applied to the cost of $110 000 000 is $1 606 000.

The total annual charge against the improvement is summed up as follows:

Interest at 4%	$4 400 000
Depreciation at 1½%	1 650 000
Taxes at 1.46%	1 606 000
Maintenance	2 000 000
Total	$9 656 000

This amounts to a cost of 1.29 cents per ton-mile, based on the volume of traffic for 1923. As the volume of traffic decreases, this cost will increase.

This charge does not include the amount paid the carriers for actually moving the freight. Such figures are not available, but H. G. Moulton, M. Am. Soc. C. E., has estimated* that a rate of 0.089 cent per ton-mile would be necessary to enable a carrier to haul coal from Pittsburgh to Cairo at a profit of 5% on its investment. Although coal constitutes by far the greater bulk of the tonnage, there is enough other material handled at a much higher rate to raise the average rate level. Further, this rate assumes a through movement from Pittsburgh to Cincinnati; but the average haul on all traffic is only 90 miles, with correspondingly greater terminal delays. These two factors would result in at least doubling the rate quoted, and the increase in cost of labor, fuel, etc., since this estimate was made, would add about 60% more to adjust it to present-day costs. These adjustments would raise Mr. Moulton's estimate to approximately 0.27 cent per ton-mile, or about one-fourth the rail rate. Of course, rates on most commodities are much higher, for there is little competition by water, and rates are based largely on railroad competition.

River carriers no longer make the rates; instead, they adjust their rates to a little below the scheduled rate of the railroads, which is based on the experience of all the railroads of the United States and must stand the test of study by the Interstate Commerce Commission, a fact-finding body. This new condition has come about through the shifting of traffic from water to railroad, and the attendant specialization of traffic on the river.

The average rate is kept down by the relatively large tonnage of coal; but making all due allowance for this, it is not probable that the average cost is less than 0.27 cent per ton-mile, if interest, taxes, depreciation, and maintenance on the floating equipment, in addition to the actual cost of operation, are properly taken into account.

Adding the cost of operation, 0.27 cent, to the ton-mile charge for improvements, gives a total ton-mile cost of 1.56 cents, which is 40% greater than the rail rate. Even this figure is not comparable to the railroad cost per ton-mile, as a ton-mile by river is not the equivalent of a ton-mile by rail. All towns

* "Waterways versus Railways", 1912.

of any considerable size on the river are also accessible by rail. Table 14 shows a comparison of distances by water and the shortest rail routes.

TABLE 14.

Place.	Rail.	River.	Ratio: river to rail.
Pittsburgh to Wheeling, W. Va.	68	90	1.32
" " Cincinnati, Ohio	326	468	1.42
" " Louisville, Ky	455	604	1.33
" " Evansville, Ind.	509	784	1.54
" " Cairo, Ill.	670	968	1.45
Total	2028	2914	1.44

In order to obtain a fair comparison of rail and river cost it is necessary, therefore, to multiply the river cost by 1.44, resulting in a cost of 2.24 cents per ton-mile by river, as against 1.116 cents by rail, or just twice as much. There are two other features which, if taken into account, would further modify this comparison to the advantage of the railroads. Coal, sand, and gravel constitute 90% of the river traffic, which is more than twice the proportion on the railroads. Railroad rates on these commodities are considerably lower than the average, and if such rates were used in this comparison instead of the average, it would result in a still more unfavorable showing for the river. Further, the rail rate of 1.116 cents per ton-mile covers revenue freight only. The traffic figures on the Ohio, as compiled by U. S. Engineers, may be based only on revenue freight, but it is probable that they include all freight moved on the river, as well as the fuel for steamboats and the materials used in constructing the river improvements. If this is true, a cost per ton-mile for river traffic, based only on revenue tonnage, would be appreciably higher. To sum up, it may be said conservatively that river transportation costs at least twice as much as railway transportation. In addition to costing twice as much, river traffic is subject to other disadvantages. It is a slow movement down stream, and much slower up stream. It is subject to interruption from the periodic floods and even ice at certain times.

River transportation is not comparable with railroad transportation in regard to service, inasmuch as it is limited as to point of loading and unloading. In the case of the railroads, the service is not thus restricted; and to this is due no doubt the development of the railroads in the river territory to the disadvantage of water-borne transportation.

This comparison shows that as a transportation facility, the Ohio River may be likened to a very crooked railroad of similar length with less than one-half the average traffic density; with a very badly balanced traffic, but with grades favorable to the heavier movement; with a roadbed which cost more than three times that of the average railroad; and operating at a total cost per ton-mile at least twice that of the average railroad. Fortunately for the

shipper on this route, he does not have to pay interest, taxes, depreciation, or upkeep on its roadway, for this expense is borne by the whole United States.

This brings us to the other phase of this discussion: How Ohio River transportation and railway transportation affect each other. When the writer began this brief study, his impression was that, in this sense, there was no relation between the two. Ohio River traffic seemed so self-contained, so independent of rail transportation, and so restricted to local traffic and to certain commodities which it can handle to advantage, that he failed to see much connection between it and transportation at large. The history of river and railroads since the latter invaded the Ohio Valley, however, shows that the railroads have had a very unfavorable effect on river traffic. On the other hand, the influence of river traffic on the railroads has been negligible, although it has resulted in favoring certain communities at the expense of others, as will be shown later.

The steamboat was invented nearly a generation before the locomotive, and water transportation was in full swing on the Ohio before the railroads arrived. As the railroads developed, however, the cost of transportation was reduced, although it was fifty years before they could seriously compete with water traffic. As this reduction continued, the railroads made serious inroads on river traffic. This reduction in cost of rail transportation, if measured by the purchasing power of the dollar, is still going on, and there is every reason to believe that it will continue indefinitely. On the other hand, river transportation has made little or no progress in that direction, and, from its very nature, will not be able to do so. The result may be noted in the trend of traffic on the Ohio. The peak was reached in 1900, since which time it has grown less at a fairly uniform rate, if the fluctuations due to varying business conditions are eliminated. This is shown in Table 15, which also shows the increase of railroad freight traffic for the same periods.

TABLE 15.

Year.	Tons of Freight.		Percentage: Ohio River of railroads.
	Ohio River.	Railroads.	
1900	14 054 322	1 081 983 301	1.30
1910	11 111 216	1 849 900 101	0.60
1920	9 869 325	2 427 622 267	0.41
1923	8 280 520	2 411 239 000	0.34

An editorial in *Engineering News* of May 4, 1916, stated that "Commerce has practically deserted most of the inland waterways. The great and controlling cause is the development of railway." This is undoubtedly true, but there are other contributing causes. The mines that can ship by river to advantage, on account of being directly on the river bank, are becoming exhausted, and the new mines that are being opened, are inland and are served by railroads.

The production of coal, per capita, is declining, not only on the Ohio River, but throughout the United States. This decrease is shown by the following information taken from the reports of the U. S. Geological Survey:

Calendar years.	Production of coal in net tons.
1913	569 961 000
1914	513 523 000
1915	511 617 000
1916	590 101 000
1917	651 399 000
1918	678 212 000
1919	546 155 000
1920	658 264 932
1921	506 395 401
1922	476 951 121
1923	657 446 000
1924	573 494 000 (estimated)

This statement shows that the production in 1924 is practically the same as in 1913. In the meantime, however, the population of the United States had increased from approximately 96 000 000 in 1913 to 112 000 000 in 1924.

The reason for this decline per capita is not difficult to find. The use of electricity has been growing by leaps and bounds, as shown by the following figures obtained from the Census Bureau:

Year.	Electricity generated, in 1 000 kw-hrs.
1907	5 862 277
1912	11 569 110
1917	25 438 303
1922	40 291 536

Part of this electricity is derived from water power, but the major part is obtained from coal. Much of the coal, however, is consumed at central locations provided with ample transportation facilities. The widening use of electricity results in less transportation of coal whether the electricity is derived from water power or from coal.

Another reason for the decline per capita in the use of coal is the rapidly increasing production of petroleum, as shown in the following figures obtained from reports of the U. S. Geological Survey:

Year.	Production of petroleum, in thousands of barrels of 42 gal.
1910	172 769
1911	220 449
1912	222 935
1913	248 446
1914	265 763
1915	281 104
1916	300 767

1917	335 316
1918	355 928
1919	378 367
1920	442 929
1921	472 183
1922	557 531
1923	725 702

The automobile is responsible for a large part of this increase, but not for all of it. The majority of the locomotives in the Southwest are now equipped for burning oil, and its use for this purpose is rapidly extending. On January 1, 1924, there were in use on the railroads of the United States 6 763 oil-burning locomotives. Still more marked is the increase in the use of fuel oil for vessels, domestic consumption, etc.

In fact, it is evident that the railroads of the United States have reached their peak of tonnage per capita. The decrease in the quantity of coal transported has been made up to a certain extent by the large quantity of materials handled for highway construction. This will probably continue for many years, but it is doubtful whether at any time it will exceed the tonnage now handled.

The relative decline in coal production will affect both river and rail traffic, but will affect river transportation more seriously, as coal constitutes about three-fourths of the Ohio River traffic, and when it goes, there is nothing to take its place. On the other hand, the railroads will have more capacity to handle higher grades of freight; they now have excess capacity, and could probably handle 25% more business than they do, without taxing their present facilities. For this reason, the declining coal traffic may cause some hardship for a time, but as the country develops, this loss will be more than made up by greater shipment of higher grade commodities.

Railroad freight rates generally are so much higher in Europe than they are in the United States that there is a wider margin for successful operation on the rivers and canals. The average revenue per ton-mile for 1913 in a number of European countries and in the United States is shown in the following tabulation, pre-war figures being chosen in order to eliminate the effect of varying conditions and exchange rates since the war:

	Cents.
Great Britain	2.23
France	1.28
Germany	1.37
Austria-Hungary	1.51
Sweden (1912)*	1.52
Belgium (1912)*	1.13
Holland	1.36
United States	0.729

Waterways have always been favored as to railroad rates for the reason that the original rate structure was based on the existing traffic and competi-

* Figures for 1913 not available.

tion. This has often resulted in lower rates to towns along the rivers. In his discussion of the admirable paper* by William M. Black, M. Am. Soc. C. E., on "Waterway and Railway Equivalents", Edward Burr, M. Am. Soc. C. E., states that:† "The waterways, however, have justified the public's confidence and willingness to spend its money on them, although they may carry little actual commerce. To use a naval phrase they have the value of a 'fleet in being' ". This may have been true at one time, but Colonel Burr has evidently forgotten the Transportation Act of 1920. This Act establishes a fair return for the railways, and authorizes the Interstate Commerce Commission to adjust rates so that they will earn such a return. This being the case, it is evident that if any community or set of communities enjoys rates lower than they would pay in the usual run of affairs, this deficiency must be made up by other communities which pay a higher rate. Colonel Burr's "fleet in being"—in this instance, the improvement of the Ohio River—serves only to protect those who dwell along the banks of the river in an advantage secured at the expense of other communities. The shipper and consumer in Columbus and Dayton, Ohio, and Indianapolis, Ind., not only help pay for these improvements, but if the improvements fulfill their purpose and stimulate river traffic, the shipper and consumer at inland points will also pay a higher rate on account of them.

It does not seem likely, however, that the completion of the improvement project will result in any considerable increase of river traffic. Coal which formerly went from Pittsburgh to New Orleans is now obtained from Tennessee and Alabama, partly by rail and partly by water. The sand and gravel business is purely local and would have existed in about its present volume if no improvements had been made. In the light of present experience, one is forced to the conclusion that the construction of these works was an economic waste. The Ohio is the one river in the United States on which there seemed to be a fair prospect of developing a large and important traffic. These great expectations have not been realized, and the writer feels it must be admitted that the experiment is a failure.

It is not probable that such an improvement would be undertaken at the present time. The attitude of the people toward the railroads has changed since 1910. The thinking part of the population now realizes that the railroads are its friends and servants, and not its enemies and would-be masters. As a people, Americans think more in economic terms than they did a few years ago. Even so, it is difficult to predict what may result from an active propaganda directed by interested parties and acclaimed by politicians who are seeking a campaign slogan.

There is a lesson to be learned from the results of the Erie Canal. In his recent Annual Message to the New York Legislature, Governor Smith stated:

"The Barge Canal, including construction, terminals, grain elevators, repairs, maintenance, operation, and payment of claims for damages, has cost the people since 1905 up to date, $191 626 000; between 1905, when the first

* *Proceedings*, Am. Soc. C. E., August, 1924, Papers and Discussions, p. 837.
† *Loc. cit.*, February, 1925, Papers and Discussions, pp. 315-316.

canal bonds were sold, up to 1924, inclusive, we have paid in interest $39 880 387, making the total cost of the barge canal to date, $230 881 014."

In the six years following the opening of the new Barge Canal in 1919, the freight averaged 1 640 481 tons. Interest on the cost of the Canal at 5% is $11 544 000, to which should be added $4 182 696 for maintenance and operation, making a total annual charge of $15 727 000, which equals $9.59 for every ton of freight handled.

In 1911, when the agitation for a deep waterway from the Lakes to New Orleans was at its height, Mr. H. G. Moulton, after describing the earlier waves of popular enthusiasm for the development of waterways, stated:

"Attention should be directed to the incessant suggestion in public discussion and in the press that transportation upon our rivers and canals should be revived. The remarkable conjunction of influences should be recalled, namely, the lingering glory of our waterways, the widespread movement for the conservation of all our natural resources, the relation of waterways to the reclamation of flooded lands, to the development of water power, and to sanitation, the supposed remarkable cheapness of water transportation, the opposition to the monopolistic tendencies of the railways, the inadequacy of the railway service to meet the country's needs, the alleged benefits of waterways even to the railways, the avowed advocacy of waterway development by railroad men, and the extensive use of waterways in foreign countries. Finally, it should be noted that, in addition to this incessant and broadside suggestion, the movement has had the active support of some economists, and of many prominent business men and politicians, who have served to give the necessary sanction of authority. Under such a cumulation of causes and combination of influences, it was utterly impossible for the average individual to weigh the various arguments which were presented or to subject them to the test of reason; and it is not surprising that a large portion of the populace should have grown excited over the matter and jumped to the conclusion that the industrial growth of the country is fundamentally dependent upon the development of water transportation."

The country is now in the midst of a similar agitation for the "Lakes-to-Ocean" waterway. It is to be hoped that a decision in regard to this gigantic project will only be reached after it has been most carefully considered in all its engineering and economic aspects, and that such a decision shall be made on the basis of the findings of the most expert investigators and not in response to an ill-informed and more or less hysterical popular enthusiasm, as has too frequently been the case in the past.

RIVER-RAIL TERMINAL

By Julian A. Pollak,* Esq.

What is the meaning of a river-rail terminal? First, it means that the river will hep the rairoads; second, it means that the railroads will help the river; and, finally, that both will help the port as well as the so-called hinterland. Every city of prominence and importance has been made in two ways—either by water, such as lakes and seaports, or by railroads. The largest cities in the United States, and those of the greatest importance, have a combination of both water and rail. New York, N. Y., Chicago, Ill., Cleveland, Ohio, Baltimore, Md., Pittsburgh, Pa., Detroit, Mich., and San Francisco, Calif., are a few examples of the wonderful combination of water and rail.

Engineers are familiar with the improvement for navigation of the Ohio River and the great plan of development of inland waterways. When this entire program is complete in the near future, the question is, what next? The answer to this question is the problem of each locality so situated that it can and will use the facilities at its disposal. For this reason the writer will confine his remarks to a definite locality—the City of Cincinnati.

Begin with a natural highway equivalent to a 10-track trunk line from Pittsburgh to Cincinnati, built at a total cost of $103 000 000 for engineering development to make it navigable for approximately twelve months in the year, barring ice and exceptional conditions. When this waterway is completed, it will connect with other waterways such as the Mississippi River which, in turn, connects with seaport, ocean, canal, and more ocean.

Locally, this means that a single city on the banks of the Ohio River begins to think in terms of New Orleans, La., Seattle, Wash., even Hong Kong, China, to say nothing of neighboring citys such as Pittsburgh, Pa., Louisville, Ky., and St. Louis, Mo. By visualizing such commercial connections, the inland neighbors in Springfield, Columbus, and Dayton, Ohio, will see that trade opportunities await them also.

It is obvious that with a highway passing the door, with money already spent on river improvements, and with trade developing in such a startling manner, that locally the citizens of Cincinnati should think about handling it. When millions of dollars are being spent on river craft, such craft should be invited to stop at this port.

One of the difficulties in the interchange of traffic between water and rail is the cost of transfer. That must be reduced to a minimum of cost and a minimum of time. It must be done in the most expeditious and cheapest manner which modern science can provide. The key to securing this minimum of time and expense in transfer is the terminal—the rail-water terminal located on the banks of the navigable river, connected by belt line with all the railroads serving the community, and provided with cranes and other modern

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facilities for transferring freight between the warehouse and the boat, or the boat and the car, cheaply and expeditiously; and it follows that this key must be provided. Who shall provide it, the United States, the States, or the political sub-divisions—the municipalities?

In order to handle this approaching situation, a river-rail transfer company has been formed in Cincinnati to connect the rail and water facilities. The first idea was to form a company which eventually would become a municipal enterprise, thus opening the Port of Cincinnati to all. Owing to the lack of funds in the City Treasury, as well as the political complications that might arise, it was decided to organize this company other than municipal and strictly on a commercial basis and for profit. The advantage of such a plan is that it stimulates the individual initiative which developed the railroads and their terminals.

The group interested plans to make this enterprise quasi-civic by encouraging the large and small shipper alike to share in the company. There are no large stock subscriptions, but it is planned to have a city-wide list of stockholders. Already the subscription list includes industrials, lawyers, department stores, advertising agencies, insurance companies, and banks—some are directly interested and others are interested purely for civic reasons. The company will be free from any monopolistic tendency, and such that any one can have any commodity handled at any time. The project, in a sense, is pioneer work, for the company has neither handled costs nor guaranteed shippers, but it has faith in water transportation, and feels that if the Congress of the United States has sufficient faith in the project to make the Ohio navigable, the citizens should have enough confidence to use such facilities.

In order to design a transfer terminal to accommodate a variety of shippers, many things must be considered before proceeding with the work. First, there is the usual incorporation to cover the business aims or business franchise; next, the location of property easily accessible to the railroads and automobile trucks. This is the first problem, because, in addition to being on the water, the facilities must be above high water for protection of machinery, storage, railroads, and streets. A high abrupt river bank will overcome the flood-water obstacle, but when the bank is abrupt, the cranes must be of ample capacity and have sufficient swing to reach the barge in pool stage as well as the top of the bank where the tracks and trucks are situated.

The next problem is a natural harbor, free from ice, and situated so that drift does not injure the barges moored for unloading and loading. In this respect valuable assistance was obtained from the Government engineers who aided by surveys and reviewed the plans. When the location and plans were submitted to the Government engineers the permits and harbor-line locations were granted. From this stage, it was possible to estimate the cost of dredging, excavation, piling, fill, and tracks, and thus work back to a first unit capacity for crane, storage, and handling.

Added to all this is the usual easements, property rights of approach, and arrangements with the railroads. The writer has only attempted to describe

briefly a few of the problems involved in locating a transfer terminal with the idea that some engineers may utilize the experience of the company in so far as it has gone.

It is felt that now is the time to go ahead so that when the 9-ft. stage is completed, facilities will be ready to handle the traffic and to point out to others that there is ample room for more companies of a similar character to do likewise.

HIGH SPECIFIC SPEED HYDRAULIC TURBINES IN THEIR BEARING ON THE PROPORTIONING OF THE NUMBER OF UNITS IN LOW-HEAD HYDRO-ELECTRIC PLANTS

A SYMPOSIUM

Discussion*

By Messrs. Edward P. Hamilton, and H. K. Barrows.

Edward P. Hamilton,† Jun. Am. Soc. C. E.—The modern propeller wheel, as developed by Mr. Nagler,‡ has many advantages over the Francis type for certain conditions. This is clearly evidenced by the large number of installations of the propeller type during the last eight years. It must be gratifying to Mr. Nagler to realize how readily what appears to be a most radical change in wheel design has come into common use.

In his paper Mr. Nagler takes exception to statement§ by Arthur T. Safford, M. Am. Soc. C. E., and the speaker, that, "a wheel of the high-speed propeller type was patented and on the market fifty years ago", and that the "Green Mountain" wheel patented by Jacob Truax in July, 1860, was the forerunner of the modern propeller wheel. It was intended to show that the "Green Mountain" wheel bears much the same relation to the propeller runner designed by Mr. Nagler that the early Francis, Swain, or Hercules runners do to the modern high-speed reaction turbine, such as the Leffel "Z" model, designed by Professor Zowski.

Fig. 57 (*a*) shows the Francis runner of 1847 alongside the Zowski runner of 1913, Fig. 57(*b*), while Fig. 58(*a*) shows the Truax "Green Mountain" runner of 1860 beside the Nagler runner of 1915 Fig. 58 (*b*). A glance at these photographs would seem to make it evident that the Nagler runner of 1915 would recognize the "Green Mountain" runner of 1860 as its forebear much more readily than the Zowski runner of 1913 would recognize its ancestor in the Francis runner of 1847.

The main structural difference between the Truax and the Nagler runner is the band which surrounds the former. Remove this and the wheels look

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* This discussion (of the Symposium on High Specific Speed Hydraulic Turbines, presented at the meeting of the Power Division on January 22, 1925, and published in August, 1925, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Asst. on Eng. Staff of Proprietors of Locks and Canals on Merrimac River, Lowell, Mass.

‡ *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 1032.

§ "The American Mixed-Flow Turbine and Its Setting", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1237.

very much alike. Fig. 59, taken from Emerson's "Hydraulics" published in Willimansett, Mass., in 1892, shows the Barber wheel, which was of the propeller type and had no band. Several other wheels of the Sixties and Seventies, such as the Blackstone and the Milliken, were built without bands around the runner. The Barber wheel is most interesting because, although the hub is large and the speed consequently low, the projections of its buckets and hub do not completely cover the area circumscribed by the bucket tips. In other words, the buckets fail to meet or overlap in the same way as those of a marine propeller or a Nagler runner.

Fig. 60 also taken from Emerson's "Hydraulics", shows the "Economy" wheel, made in Indianapolis, Ind. The two propeller runners have no bands. Note, also, the comparatively flat angle of the blades.

Fig. 61, taken from Bélidor's "Architecture Hydraulique", published in Paris in 1737, shows a type of wheel very common in those days in Southern France on the River Garonne and its tributaries. It was used on low falls, and was set in a masonry scroll of very much the same proportions as the wooden ones used a hundred years ago in America. Note the marked narrowness of the seven blades and their failure to overlap, leaving one-quarter or more of the total runner area entirely open in an axial direction.

Fig. 62, taken from Truax's 1879 circular, shows the wheel and the form of casing that he recommended. He advised that his wheels be set in wooden rather than iron spiral casings, which, he said, cost more and were less adaptable to varying conditions.

The following extracts from Jacob Truax's "Green Mountain" circular of 1876 show a striking resemblance between the claims made for the "Green Mountain" wheel and those made for the modern propeller wheel:

"To Mill Owners.—Please do not hesitate, for I am offering you the only wheel that is adaptable for the low and common falls, and selling a wheel of the same power, using as little water, for less money than any other wheel now in use.

"I am having the most gratifying assurance from my patrons of the unequaled efficiency or superiority of this wheel over all other wheels, for common or low falls.

"This wheel has been in practical use over fifteen years, and has proved to be one of the best water wheels now in use."

"* * * the Green Mountain Water Wheel is a great money saver, for mill-owners that have low falls. One-half of the money that is laid out on a 5-ft. fall by Leffel and Risdon, or nearly all other wheel manufacturers, is saved by the use of the Green Mountain Wheel, and one-fourth on a fall of 9 to 10 ft. There is no material difference in the use of water for the same power on the same falls, but the capacity of the Green Mountain Wheel is such that much less diameter wheel is necessary to use the same cubic feet of water per minute. It being so much smaller it rotates much faster, and is impeded but little by back-water compared with other wheels. It will not wear out or break or clog with anchor ice or flood trash. Its smallness is the cause of its cheapness."

There follows a list of thirty-three installations of "Green Mountain" wheels on falls of between 4 and 10 ft., the average being 6.7 ft., after which the circular continues:

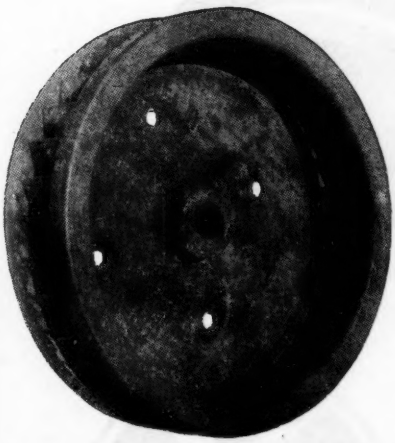


FIG. 57(*a*).—ORIGINAL FRANCIS RUNNER.

FIG. 57(*b*).—MODERN LEFFEL "Z" RUNNER.

FIG. 58(*a*).—GREEN MOUNTAIN RUNNER.

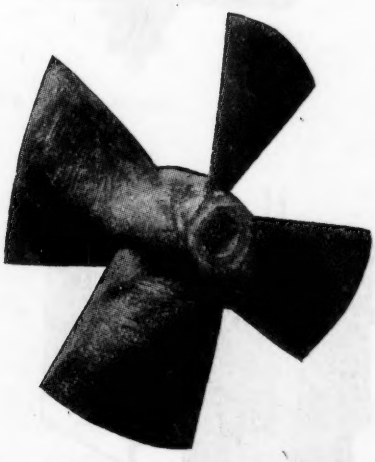
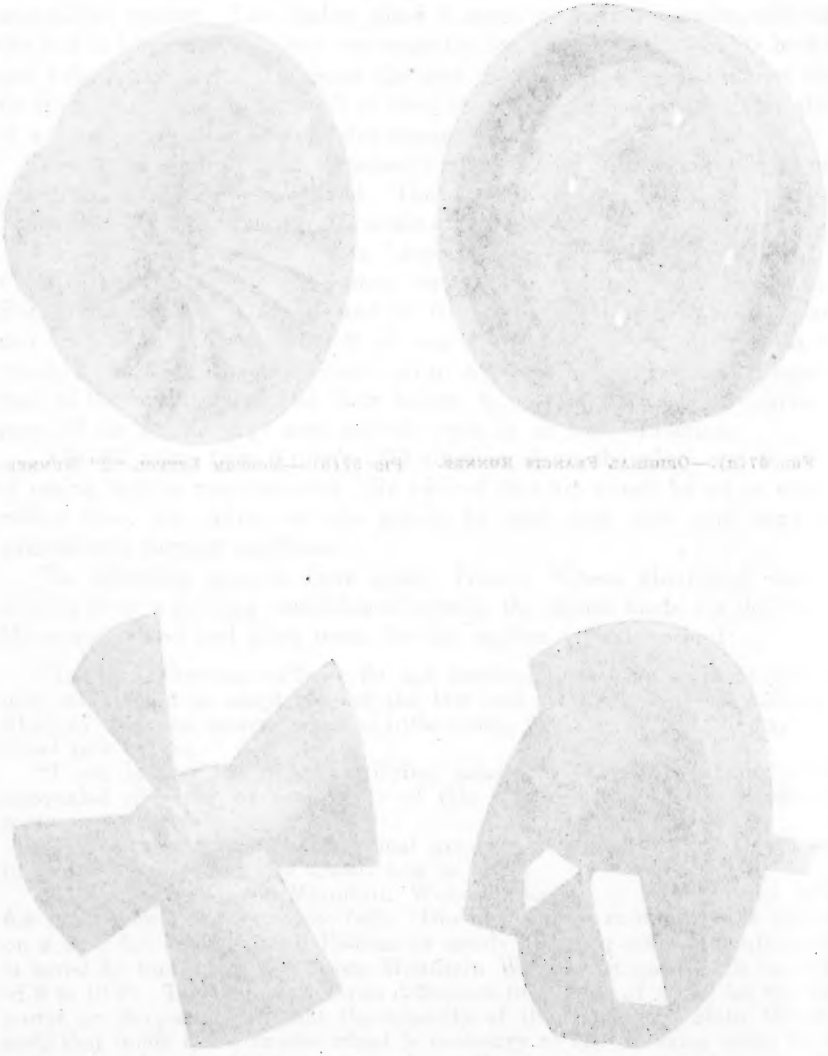


FIG. 58(*b*).—NAGLER RUNNER.



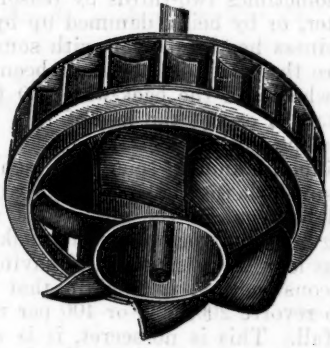


FIG. 59.—THE BARBER WHEEL.

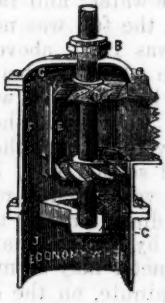
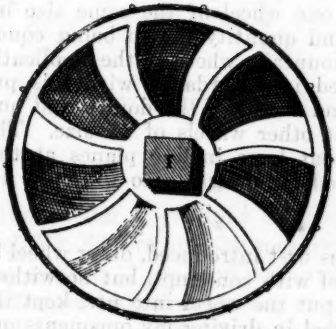


FIG. 60.—THE "ECONOMY" WATER-WHEEL.



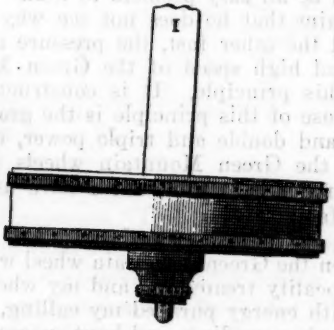


FIG. 61.—FRENCH TUB WHEEL RUNNER.

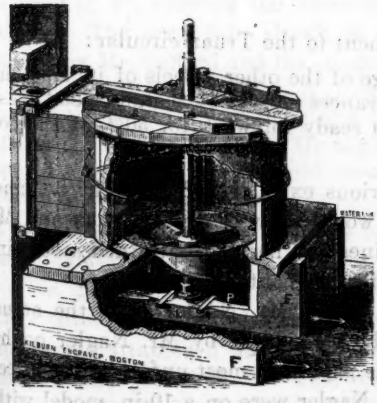


FIG. 62.—TRUAX WHEEL IN WOODEN CASE.

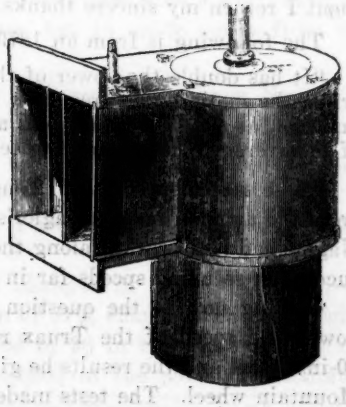


FIG. 63.—TRUAX WHEEL IN IRON CASE.

"These are a few of the low falls that my wheel is working on, and the falls are sometimes reduced one-half, sometimes two-thirds by reason of low water, or back-water caused by high water, or by being dammed up by ice below the mills, or tide-water; and fair business has been done with some of the above wheels when the fall was not more than 2 ft. They have been run when the head-water was 12 ft. above the wheel and the back-water 10 ft., leaving but 2-ft. fall, the wheel not being retarded except the loss of fall. This result is not attained by any other wheel in the world.

"It is a well established principle among scientific men that velocity doubles every four times the pressure, whether it be steam, air or water. This being a well settled fact, it will be seen, in the absence of any additional pressure, the velocity in a rotating body may be doubled by making the incline at the mean diameter four times as long as it is deep, and having thus doubled the velocity by this mechanical construction, it is plain that a water wheel of equal diameter may be made to revolve 200, 300, or 400 per minute, instead of 100 per minute, on the same fall. This is no secret, it is a plain, simple principle, and ought to be understood by every wheel builder. If the wheel builder understands this little principle or rule, he will not wonder, but say Eureka. The fact that a 4-ft. wheel may rotate 200 per minute on a fall of 6 ft. will be an easy problem to him. This principle understood, and he will not exclaim that he does not see why one wheel of the same size may run slow and the other fast, the pressure and quantity drawn being equal. The power and high speed of the Green Mountain wheel is the application and use of this principle. It is constructed in accordance with this principle, and the use of this principle is the grand secret of the double and quadruple motion, and double and triple power, of other wheels of its size. There are none of the Green Mountain wheels that have incline planes at the mean circumference of 4 to 1, but I have used these figures to illustrate the rule, and the fact.

* * * *

"When the Green Mountain wheel was first introduced, other wheel builders in this locality treated me and my wheel with contempt, but notwithstanding this I with energy pursued my calling, put the wheel into use, kept the hammer of logic pounding, and have succeeded in driving my opponents quite out of the field. Many of them now acknowledge the usefulness and superiority of the Green Mountain Water Wheel, saying that if a man has a low fall, and fails with the Green Mountain wheel they may stop trying. For this compliment I return my sincere thanks."

The following is from an 1879 supplement to the Truax circular:

"It has double the power of the average of the other wheels of its size, and triple the motion, and free from the hindrances caused by ice and flood trash, and can, as a rule, be placed in a position ready for work for 50% of the cost of the average of other good wheels."

These statements placed alongside various extracts which might be made from the articles on Mr. Nagler's runner would show that Mr. Truax and Mr. Nagler had both worked along the same lines and had both attained a striking success in securing speeds far in excess of other contemporary types.

Coming now to the question brought up by Mr. Nagler as to the actual power and speed of the Truax runner, the tests made by Mr. Nagler on his 10-in. model and the results he gives in his paper are most unfair to the Green Mountain wheel. The tests made by Mr. Nagler were on a 10-in. model without any scroll-case or draft-tube and under a head of 12.90 ft. The Truax wheel tested in Lowell, Mass., was an actual 32-in. runner that had been in use

for many years in a mill in Northern New York. The results of the two tests are entirely different, as shown in Fig. 64. It is very interesting to note that the uppermost of the speaker's curves reverses and turns back toward the origin in exactly the same manner as the propeller-wheel performance curves shown by Professor Moody*.

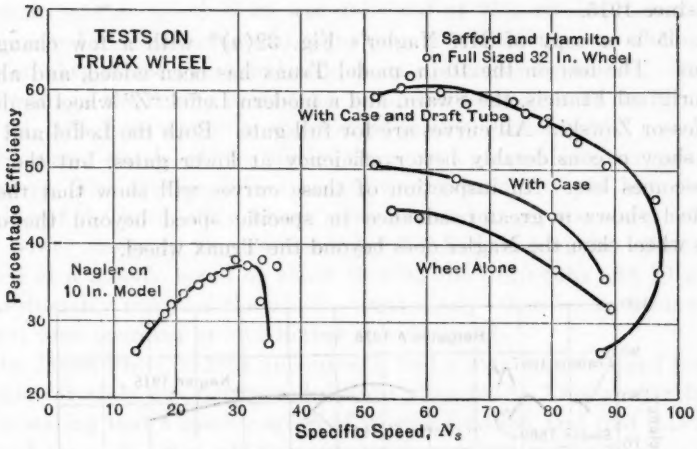


FIG. 64.

In the closing discussion† of their paper on "The American Mixed Flow Turbine" Mr. Safford and the speaker pointed out the unfairness of the Nagler tests, on account of the head used and of the omission of the casing.

Mr. Nagler feels that the use of a scroll-case and a draft-tube by Mr. Safford and the speaker made an essential addition to the Truax wheel. The Truax patent of 1860 specified both. The present casing was built to conform to the dimensions for a 32-in. wheel given in the 1876 circular from which was taken Fig. 63, showing the wheel in an iron scroll-case with a draft-tube attached. The draft-tube shown is not flaring, but to quote again from the paper‡ by Mr. Safford and the speaker:

"It is interesting to note that 'W. W. T.,' writing of the 'Green Mountain' propeller wheel, realized that, with a wheel of this type, a draft-tube would be of greatly increased value. He says, 'If Mr. Boyden would put his diffuser on one of these wheels, we would promise him a gain of more than 3 per cent. It would be more nearly 50.'"

The draft-tube used was far from ideal, and it actually added only about 19% to the power.

Mr. Nagler's Table 2§ requires certain changes in order to be correct. He credits the Truax wheel with a maximum of 61% efficiency and 60 specific speed. It attained a maximum specific speed of 98 in the test made by the speaker and Mr. Safford. This was not reached at the point of maximum

* *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, pp. 1015 *et seq.*
† *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1922), p. 1346.
‡ *Loc. cit.*, p. 1286.
§ *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 1035.

efficiency, but at a considerably lower point (37%); at a speed of 97, the efficiency was 46 per cent. It is interesting to note that while Mr. Nagler tabulates the values of efficiency and specific speed for the Truax for the same point, he uses two different points for his own wheel giving high values for both. In his original wheel he secured a specific speed of 129 at 87% efficiency, rather than the 150 of Table 2. Of course, his wheel has been greatly improved since 1915.

Fig. 65 is a copy of Mr. Nagler's Fig. 32(*a*)* with a few changes and additions. The test on the 10-in. model Truax has been added, and also tests on the original Francis, the Swain, and a modern Leffel "Z" wheel as designed by Professor Zowski. All curves are for full gate. Both the Leffel and Nagler wheels show a considerably better efficiency at lower gates, but the specific speed becomes less. An inspection of these curves will show that the Leffel "Z" wheel shows a greater advance in specific speed beyond the original Francis wheel than the Nagler does beyond the Truax wheel.

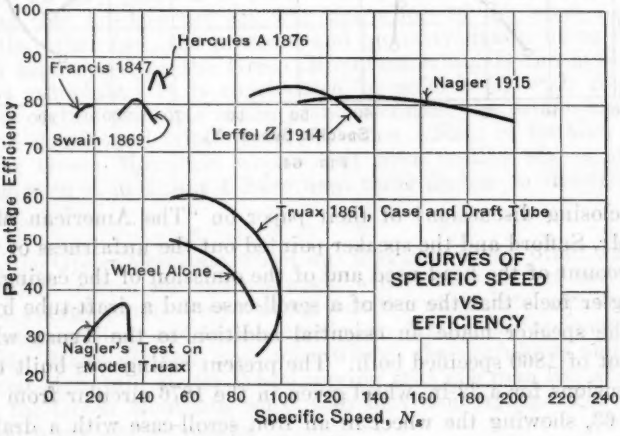


FIG. 65.

It is granted that the efficiency of the Truax wheel was low. It must be remembered, however, that its efficiency compared very favorably with that of its contemporary wheels. By 1860 a great many small crude turbines were beginning to come into use, but the usual wheels in the little mills in the northeastern part of the United States were the "flutter" and Rose wheels in saw-mills, and tub or scroll wheels in grist-mills and other small shops. Some undershot and overshot wheels were used, of course, but ice conditions were such as greatly to restrict their use. The "flutter" wheel probably gave an efficiency of from 15 to 25%, the Rose wheel considerably more, perhaps even as high as 60%, while the tub and scroll wheels ran from about 20 to 40 per cent. In the fall of 1923, in connection with some work at Calais, Me., the speaker ran a test on an extremely good example of tub wheel. An efficiency of 31% was obtained on the jack-shaft, equal to about 36% on the wheel-shaft.

* *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 1036.

Until the latter part of the Nineteenth Century the efficiency of a water-wheel was of comparatively little importance; power was what was required. At the time the Truax wheel appeared, aside from a few tests in Philadelphia, Pa., in Lowell, and in France, the testing of water-wheels was almost unknown. It was not until about 1870 that systematic testing began.

The Truax wheel delivered the required power at a high wheel speed. Its maximum specific speed of 98 was delivered at 37% efficiency, but it gave a specific speed of 90 at somewhat more than 50% efficiency, an efficiency which exceeded that of the great majority of its contemporary wheels. Testing the wheel without the flaring draft-tube, as undoubtedly most of the Truax wheels were set, still gives a specific speed of 80 at 43% efficiency.

The water-wheels of the Sixties were not operated at their point of best efficiency. The average miller or millwright hardly would have known the meaning of the word. The wheels were operated to get the power. The maximum power shown in the test made by Mr. Safford and the speaker was developed at a specific speed of about 80 with the draft-tube and 72 without it. Unquestionably many of the wheels, particularly when under back-water conditions, were operated at still higher specific speeds.

The Truax wheel of 1861 undoubtedly had a high specific speed for its time, and this certainly was not exceeded until after 1910. The speaker feels justified in stating that a specific speed of 90 was attained, and that this speed was attained at an efficiency which was more than ample to justify the commercial use of this wheel. Accordingly, Mr. Nagler's chart of specific speed (Fig. 32), should be modified as shown in Fig. 66.

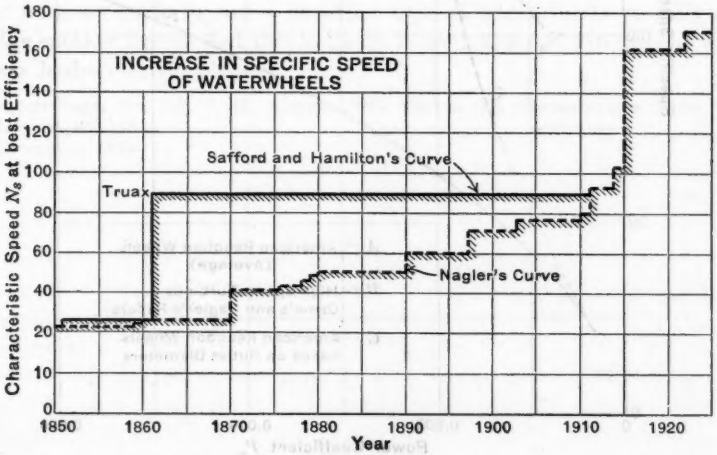


FIG. 66.

Mr. Nagler has developed a crude wheel of sixty years ago into a commercial type of wheel that satisfies present-day requirements of efficiency and mechanical construction, and great credit is due him for this. The speaker's only contention is that Mr. Nagler has not invented a new type of water-wheel, but that he has improved an existing type.

H. K. BARROWS,* M. AM. SOC. C. E.—The speaker has been much interested in the papers† presented before the Power Division; the data, particularly for high-speed turbines, furnish information of value in the study of this type of wheel.

For comparison with wheels of lower specific speed the speaker has prepared Fig. 67, on which is plotted specific speed, $N_s = \frac{N\sqrt{HP}}{h^{\frac{5}{4}}}$, against the power coefficient of the wheel. The latter is P_u in the equation,

$$H P = P_u D^2 h^{\frac{3}{2}}$$

or it is the power delivered by a 1-in. wheel under a head of 1 ft.

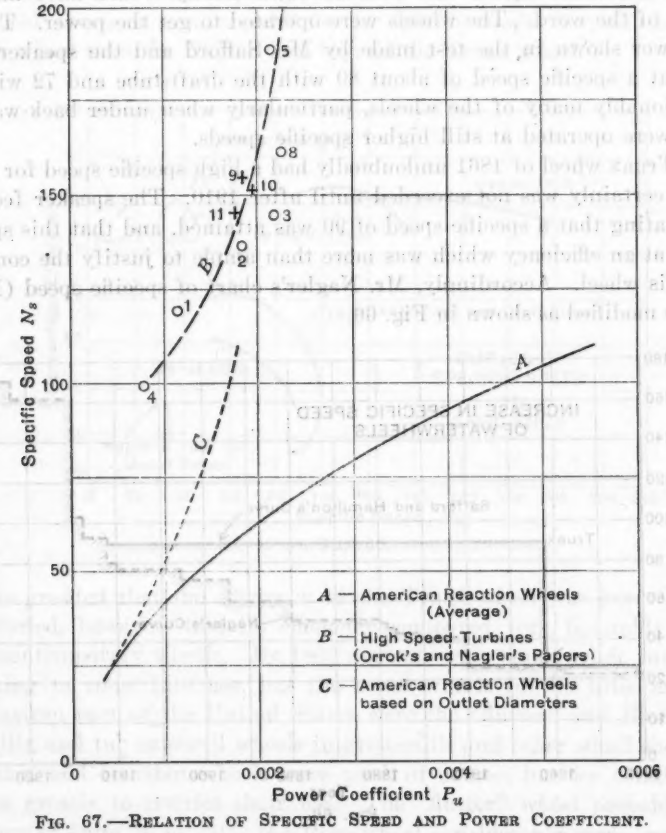


FIG. 67.—RELATION OF SPECIFIC SPEED AND POWER COEFFICIENT.

Curve *A*, Fig. 67, is plotted from data of a large number of American reaction wheels of different makes. It is an average curve, hence individual wheels will depart somewhat from it in both directions.

* Prof. of Hydr. Eng., Mass. Inst. Tech.; Cons. Engr., Boston, Mass.

† *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 999.

Curve *B* is an approximate curve based on the data given by Mr. Orrok in Table 1* (numbering the wheels for convenience from 1 to 8, inclusive), and by Mr. Nagler in Figs. 6† and 8‡ (the latter numbered 9 and 10, respectively, on Fig. 67). The point numbered 11 is for the Nagler runner at the Plainwell, Mich., plant.

At first glance it would appear that the propeller type of wheel delivered much less power for a given diameter and head than the ordinary reaction wheel. This is due, however, largely to the different manner in which the wheel diameters are measured. With the propeller type the nominal wheel diameter is approximately equal to the outlet diameter. With the ordinary reaction wheel the nominal diameter is measured at the entrance and is considerably less in amount than the outlet diameter. If outlet diameters are used in plotting the $N_s - P_u$ curve for reaction wheels as shown in Curve *C*, Fig. 67, it will be seen that the two types of wheels are not greatly inconsistent in this respect. For specific speeds approximately 100 to 110, the propeller type still shows a somewhat less power coefficient than the ordinary reaction wheel.

Considering the turbine as a form of orifice, it will be seen that for a given area at the outlet there should be a similar discharge and power for any given head (at, say, full gate) provided efficiencies are the same. As the propeller type is usually about 5% less in efficiency than the ordinary reaction wheel the power coefficient of the former is bound to be somewhat less.

The relatively sharp peak of the load-efficiency curve of the propeller type of wheel renders its use less advantageous where the wheel must frequently run at part load, as is usually the case. While under some circumstances it may be practicable to run a propeller type of wheel fairly steadily at best gate, the speaker agrees that this is likely to be difficult of attainment with the ordinary hydro-electric plant.

* *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 1004.
† *Loc. cit.*, p. 1038.
‡ *Loc. cit.*, p. 1039.

THE ST. LAWRENCE WATERWAY TO THE SEA

Discussion*

By Messrs. H. de B. Parsons, H. C. Sadler, F. P. Williams, Thomas H. Hogg, and L. C. Sabin.

H. de B. Parsons,† M. Am. Soc. C. E.—The biggest engineering project now under discussion concerns the development of the St. Lawrence River. The project contemplates the canalization of the river so that ocean-going vessels may enter the Great Lakes, and the development of hydraulic power between Montreal and Lake Ontario.

Much has been written and spoken about the St. Lawrence project, but a large proportion of it has been propaganda designed to awaken popular fancy. The canalization of the St. Lawrence and of the rivers joining the Great Lakes, coupled with hydraulic power development, is such a gigantic undertaking that the problem requires intensive study based on facts before any sound opinion can be reached. The engineering difficulty of damming an immense river, situated in a cold climate and lacking a low-water period to facilitate coffer-dam construction, is not the only problem. A greater difficulty is that of correctly conceiving future conditions of finance and trade, in order that the completed project may be of greatest National benefit without excessive cost.

The northern part of the Middle West is a "big" country, it has had a "big" growth—even phenomenal—and it is easy to comprehend why its people should be interested in a "big" undertaking. Projects of great magnitude always have to go through a long period of discussion and duplication of investigation. Probably it is right that they should be subjected to such scrutiny, as to remedy an oversight, error of judgment, or mistake in application would be most costly once the works were started. Mistakes and errors usually are attributable to a lack of clear understanding and appreciation of underlying principles. It is easy to sink capital in an enterprise which will not pay a return on the investment, whereas it is difficult to forecast future requirements in an age of progress, such as the present.

This paper covers the chain of the Great Lakes as well as the St. Lawrence River. The speaker concurs with the author regarding benefits that would accrue from a proper regulation of the lake levels. He is not able to follow the author's assumption that ocean-lake transportation cost will approximate the

* This discussion (of the paper by Francis C. Shenehon, M. Am. Soc. C. E., presented at the Fall Meeting of the Society, Detroit, Mich., on October 23, 1924, and published in September, 1925, *Proceedings*), is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., New York, N. Y.

present cost of lake freight charges, as this conclusion has not been fully accepted by disinterested parties. He is not in accord with the author on costs of construction, as mentioned in the paper, because he believes they will be greater than stated, and also because the costs of deepening lake harbors and of building harbor protection works and dock facilities have not been given sufficient consideration. Nor can he subscribe to the conclusion that hydraulic power development along the waterway will subsidize canalization, because it is by no means certain that the Governments of Canada and of the United States, after constructing such power developments, can operate or lease them for the purpose of subsidizing navigation. In brief, he believes that the paper is an excellent introduction to his argument, namely, that the whole subject should be fully considered by an International Commission composed of disinterested and unbiased engineers.

The speaker always has believed that any project which benefits a large section of a country and a large proportion of its inhabitants is of advantage to all, even to those who cannot make use of the project nor receive any direct service from it. Prosperity or depression in one section is reflected in other sections, although it may not be evident at the time. While people in the great divisions of the United States have different aims, different interests, and different viewpoints, they are linked together by economic consideration in such a manner that their common welfare is of greater import than sectional benefit.

The entire project is in reality two projects linked together, namely, first, a ship canal through the St. Lawrence to Lake Ontario, utilizing such power as can be generated at the dams, remodeling the Welland Canal, and deepening interlake rivers and lake harbors by dredging and by lake-level regulation works; and, second, an hydro-electric power development between Montreal and Lake Ontario, using the ponds above the dams for navigation purposes, while the remodeling of the Welland Canal and the deepening of the interlake rivers and lake harbors would follow the use made by ship navigation.

Briefly, the questions are: Should navigation or power generation be the primary factor in the development of the river between Montreal and the outlet of Lake Ontario, and, which primary consideration would be the better for the States and the Dominion? These two primary factors, although closely related, should be kept separate. Dams best suited for canalization necessarily would not be the same in height or location, as those where power generation was the dominating consideration.

The St. Lawrence is the natural drainage outlet of the Great Lakes. These lakes form immense storage reservoirs, so that the yearly flow of the river is remarkably even. The annual variation of the surface of Lake Ontario is only a few inches, with a maximum at long intervals of about 5½ ft. The uniform flow of the river is a condition favorable for power development, as it will permit the use of units of large capacity with a steady ouput. In this respect the St. Lawrence is unique, its normal flow being greater than that of any other river on which hydraulic power can be developed advantageously.

The normal flow of the St. Lawrence bears a relationship to the maximum flood and to the minimum discharges of other large rivers approximately as shown in Table 5.

TABLE 5.—Relation of Normal Flow of the St. Lawrence River to the Maximum Flood and the Minimum Discharges of Other Large Rivers.

	Maximum flood discharge.	Minimum discharge.
Susquehanna, at McCall Ferry, Pa.	One-fourth	100 times greater
Mississippi, at St. Paul, Minn.	One-third	10 " "
Nile, at Assuan Dam, Egypt	Equal	10 " "
Connecticut, at Hartford, Conn	Equal	160 " "
Delaware, at Lambertsville, N. J.	One and one-half	190 " "
Hudson, at Mechanicville, N. Y.	Twice	200 " "

This comparison is more favorable to the St. Lawrence than is apparent from the data given, because the maximum flood discharges last only a few hours whereas the flows approach the minimum for an appreciable part of each year.

This remarkably uniform natural flow can be improved by regulation works near Lake Ontario. It has been estimated that the flow can be controlled between 210 000 and 280 000 sec-ft. Such regulation would be of great benefit to both navigation and power development.

The best available figures on flow of the St. Lawrence River, as measured near Ogdensburg, N. Y., are:

Maximum discharge	320 000	sec-ft., or	152%
Mean "	210 000	" " "	100%
Minimum "	191 000	" " "	91%
When ice-blocked	185 000	" " "	88%
High water in Lake Ontario (1870), in feet			248.95
Mean level of Lake Ontario (1860-1923), in feet			246.15
Extreme low water in Lake Ontario (1895), in feet			243.41
Discharge at mean level, in second-feet			240 000
Increase per foot rise of lake, in second-feet			21 500
Entire area of drainage basin, in square miles			298 080
Of which the water surface is, in square miles			95 160

The area of the drainage basin is divided between the United States and Canada, as follows:

In the United States:	
Water	60 950 sq. miles
Entire	176 050 sq. miles
In Canada:	
Water	34 210 sq. miles
Entire	122 030 sq. miles

The only tributary of any size is the Ottawa River, part of which enters Lake St. Louis, just above Montreal. Although this river has a wide range of flow, its effect on the general project is of minor importance, and would only affect hydraulic works near Montreal.

Some international discussion has taken place in regard to the 10 000 sec-ft. required by the City of Chicago, Ill., for its Drainage Canal. However, Lake Michigan is entirely within the United States and its basin has an area of 69 040 sq. miles. If it is assumed that the mean run-off is the same per square mile as for the entire basin of the St. Lawrence, namely, 0.8 sec-ft., then the Lake Michigan Basin contributes to the general flow 55 232 sec-ft., or 5½ times the quantity that Chicago is asking permission to take. The lowering by this taking of the general water surfaces of the lakes below Michigan, however, is a condition that affects Canada as well as other places in the United States besides Chicago. The surface levels of the lakes (except Superior) appear to be lowering. This may be due to deforestation and to deepening of the channels connecting the lakes. This subject should be officially studied. If lake-level regulation works are required, they should be built at once, as the gradual shallowing of channels and harbors is a serious menace to navigation.

The Canadian Provinces of Quebec and Ontario have an area of about 563 700 sq. miles and a population of about 5 500 000. The States bordering on the Great Lakes (excluding New York and Pennsylvania, because they would benefit but little by the canalization) are Minnesota, Wisconsin, Michigan, Illinois, Indiana, and Ohio. These six States have an aggregate area of about 332 400 sq. miles, or about 11% of the total area of the United States, and a population of about 22 500 000 or about 22% of the total population. If the grain-growing states of Kansas, Missouri, Nebraska, Iowa, South Dakota, and North Dakota, are included as being benefited by the canalization, then the area mentioned would be increased to 765 860 sq. miles and the population to 32 900 000, which would be 25% and 31% of the corresponding figures for the entire United States.

As years pass, this nation's demand for foodstuffs will become greater, and less surplus will be available for export. Possibly, the Dominion will have surplus grain for export when that of the United States is required for home consumption.

Even under most favorable conditions only a part of the grain and other export business would seek an outlet *via* the St. Lawrence, and the question is "What part?". The business is largely bulk freight. Return cargoes would be largely package freight, requiring different conditions for handling.

Going from Lake ports *via* the St. Lawrence rather than *via* New York the saving in distance to Liverpool is about 450 miles and to Southampton about 225 miles. There is no saving in distance to many foreign ports. Physical disadvantages—ice and fogs, dangers due to narrow channels and locks, and increased insurance—would offset the gain in distance. There would be, however, a saving in transshipment, and possibly a cleaning of ship bottoms, as barnacles and other sea-water growths would have a tendency to drop off in the fresh water of the lakes. The question then arises, "Would the

traffic make it worth while for ocean-going vessels to navigate the St. Lawrence?". The distance from the Atlantic Ocean to Montreal, *via* Cabot Strait, is approximately 797 miles; and from Montreal to Lake Ontario, 182 miles. The approximate distance from Chicago to the ocean is 2 040 miles, *via* Cabot Strait on the south of Newfoundland, and from Duluth, 2 139 miles. These distances would be increased about 200 miles *via* Belle Isle Strait on the north of Newfoundland.

The speed of vessels using the canalized river probably would not exceed 6 miles per hour, while delays at the locks, and loss of time through fog would diminish the average speed below this limit for transit from the ocean to Lake Ontario. Assuming that loss of time by reduced speed and delays at locks would be compensated by increased speed while traversing the open water of the Great Lakes, a vessel would require about 11 to 12 days for passage from the Atlantic to Chicago or Duluth.

The St. Lawrence River is closed to navigation by ice about four months each year. Practically the foreign shipping season would be about seven months, as ocean ships would leave port before there was danger of closing and would not enter until after the river had been opened.

The lake trade has developed a wonderfully efficient type of vessel. These vessels can carry about three times the tonnage on the same draft as a sea-going vessel, and their large hatchways permit rapid loading and discharging of bulk cargo like grain, ores, or coal. It is not uncommon for a lake freighter to discharge 10 000 tons of iron ore in 3 hours with four 15-ton electric buckets, each operating about once a minute. Lake vessels can load in even shorter time. This speed makes a quick "turn around", which is an economical feature for marine transportation. Ocean navigation is more expensive than lake navigation—a larger crew is required (about 60% larger), inspection regulations are more stringent, surface condensers are required in place of jet condensers, large hatchways are unsuitable for ocean service, and sea-going vessels cost per ton capacity from two to three times that for lake steamers. To-day's lake freight charges per ton-mile, therefore, are not applicable to vessels that are suitable for a trans-oceanic service. It might be possible to develop a new type of vessel—a combination, so to speak, between sea-going and lake types—but could such specially designed vessels afford to be laid up for more than one-third of each year while the waterway was ice-bound?

The St. Lawrence below Montreal has a minimum depth of about 30 ft. Between Montreal and the outlet of Lake Ontario, the river is navigable in part, and Canada has a series of canals about 14 ft. deep around natural obstructions. The Welland Canal has a depth of about 25 ft., with 30 ft. on the lock sills. The Detroit River has been excavated in rock to a normal depth of about 23 ft. The lake harbors have different depths, in general, from 16 to about 22 ft.

Before undertaking the canalization of the St. Lawrence, primarily for the purpose of navigation, it will be well to make sure that ocean-going ships will patronize it. As the power possibility is great, it would seem a pity to minimize it for a canalization project that was not founded on sound judgment. There is little doubt that the full power of the river is needed, or soon will be needed,

especially in the United States. The question to be settled is: "Should not the river be developed to best advantage for power generation, and then make use of the pondages for navigation?".

The St. Lawrence water powers are not equalled by any river in North America. Its nearness to a power market is undisputed. There is little danger that private interests alone will benefit. The Government will safeguard its rights, and public service commissions will have a voice in regulating prices and in determining the manner in which current is sold. There can be no question that the cost of development by private enterprise will be less than by the Government. As interest on the cost forms one of the largest items of annual charges, it is clear that the lower cost will make cheaper rates for power.

Rated power of prime movers in the United States is growing more rapidly than the population; in other words, there is more horse-power per capita now than formerly, and there will be still more in the near future. The Federal Power Commission has compiled data as shown in Table 6.*

TABLE 6.—DATA RELATIVE TO POPULATION AND POWER CONSUMPTION IN THE UNITED STATES, 1905 to 1930.

Year.	1905.	1910.	1920.	1925.	1930.
Population	85 000 000	92 000 000	109 000 000	113 000 000	119 000 000
Power consumption, total rated horse-power	20 000 000	26 000 000	42 000 000	50 000 000	59 000 000
Rated horse-power, per capita	0.235	0.282	0.385	0.442	0.495
Increase in percentage	.....	20	64	89	110

The figures given in Table 6 are significant. The rated power of prime movers per capita has increased in 20 years (1905-25) about 89%, and will increase in 25 years (1905-30) about 110 per cent.

The figures in detail indicate a decreasing tendency in horse-power per capita generated by manufacturers, and a rapid increase in rated capacity for electric railways and central stations. The figures show the importance of large central stations for the generation of power, with their inherent economies, and the demand that will be made on the St. Lawrence for the utilization of its power.

The chief link of the project is the 182 miles between Montreal and Lake Ontario. As the lake is about 247 ft. above mean sea level, and as the river at Montreal varies in depth from about 16 to 23 ft., the gross available fall is about 224 ft. Of this reach of the river, 113 miles, with a fall of 93 ft., forms the International Boundary, while 69 miles, with a fall of 131 ft., is entirely within the Canadian border. In brief, 62% of the distance and 41½% of the power is divided equally between the United States and the Dominion; and 38% of the distance and 58½% of the power is owned entirely by the Dominion. Therefore, the United States, or more correctly speaking

* "Super-Power and Its Public Relations", by Hon. Herbert W. Hoover, *Military Engineer*, July–August, 1924.

the State of New York, owns 20¾% of power, while the Dominion owns 79¼ per cent. The State has riparian rights covering 62% of the total distance; while the Dominion has rights covering 100 per cent.

On the basis of a mean flow of 210 000 sec-ft., a total fall of 224 ft. between Lake Ontario and Montreal, and an efficiency of 70% (to allow for mechanical losses and losses of head that cannot be recovered), there would be 3 741 000 potential h.p. This estimate is subject to refinement by load factor, diversity factor, transmission-line losses, conditions governing a super-power system, and loss of water due to lockages by passing ships. Regulation works would increase the available horse-power, as the lakes would be storage reservoirs.

Of the total stated, the Dominion, with a fall of 131 ft., has about 2 188 000 potential h.p. The power that lies on the International Boundary has a fall of 93 ft., which would give 1 553 000 potential h.p., of which the State of New York (or the United States) would own one-half.

There would have to be an agreement between the United States, the State of New York, and the Dominion of Canada, before any construction work were undertaken, either for canalization or power development. If the two National Governments equally divided the cost of construction and cost for flooded lands, the United States would have to pay for deepening its own lake ports and harbor protection works, as well as its share of blasting and dredging the interlake rivers and canals. If the canalization were made for 30-ft. draft and the full hydraulic power developed, the total cost to both Governments might exceed $1 000 000 000, without any allowance for transmission lines.

There are many vital questions to be settled, such as, what shall be the minimum depth of canal? Shall it be for vessels of 21-ft., 25-ft., or 30-ft. loaded draft? Where should the dams be located? What is the geologic structure at the proposed sites? What is to be size for locks? Should lake-level regulation works be constructed? What should be the rated capacity for generators? What load factor can be expected? What would be the cost?

The speaker believes that both Governments should unite in the appointment of an International Commission, composed of the highest grade of engineers, which should outline its recommendations in interim reports. These interim reports should be published so that constructive criticism might be received. When its final report is accepted, and an agreement between the Governments is reached, the works could be started. If a "plan and policy" for a comprehensive development of the St. Lawrence were adopted, there would be no good reason why part could not be done by the Governments and part by private interests, should such a division be deemed the most economical and the quickest procedure.

Let the investigation be full, because ordinary propaganda will not finance a great undertaking. Facts alone must guide those seeking the solution of a complicated problem.

H. C. SADLER,* ESQ.—In all large undertakings, especially those of a public character, numerous and conflicting interests are bound to be involved. When, in addition, projects that affect international as well as sectional interests are introduced, the field for discussion and criticism is expanded still further. Therefore, in the case of the Great Lakes-St. Lawrence Waterway, it is only natural to expect not only criticism, but opposition, and from its very nature, opposition of a somewhat violent character.

Although it will be generally conceded that certain points are debatable, on account of the mass of facts which can be summoned to support each side of the controversy, it is necessary that, in order to receive any consideration in the argument, the facts as stated should be true. Otherwise, the whole fabric, although for a time appearing to be substantial, must eventually collapse, as its foundation rests on a quicksand of imaginations or mistakes.

A sage of old time once said, "Would that mine enemy would write a book", and in connection with the project under discussion, many books, pamphlets, reports, etc., have appeared which contain arguments against the proposition. In this mass of literature one finds numerous statements which, to say the least, are misleading and others that are absolutely untrue.

In this discussion the speaker proposes to deal simply with matters affecting ships and their operation, and leave the other points of controversy, which involve power developments, international and sectional questions, to those whose training and experience qualify them to pass an opinion; if this method had been followed in the past, the amount of chaff in the wheat of the literature would have been reduced considerably.

Although some of the speaker's statements may have appeared before, they will bear repetition, first, because they have been somewhat lost in the mass of the literature; and, second, because such repetition may help to brand as utterly false certain other statements that may have passed muster as the truth.

Perhaps one of the most important points at which those who are interested in the questions relating to the engineering features and those of transportation meet, is in the question of depth of water in the channels and canals. This ultimately affects the draft available for ships. Rather than go into a mass of figures and technicalities as to sizes of vessels, the speaker will present the actual facts of overseas transportation, to show what the world has done and is doing to-day.

Every maritime country in the world has several ports that may be classified as "major" and "minor". An examination of "Lloyd's Register of British and Foreign Shipping" will show the depth of water available in every part of the world. With comparatively few exceptions this depth varies from about 30 ft. to 24 ft. for the major ports and from 24 ft. to 14 ft. for the others. All these ports are doing business, so it is somewhat hard to reconcile with the actual facts such statements as the following: "24 ft. is the minimum draft that can be operated profitably", or "there are no vessels * * * profitably operated in ocean service drawing as little as 18 ft.", or "all sizable ships engaged

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in foreign commerce draw in excess of 30 ft.", or, again, "the average draft of ocean-going vessels is from 30 to 35 ft."

The great mass of the world's commerce is not carried by the *Leviathans, Majestics,* etc., but by the cargo or tramp steamer, and as a matter of fact more than 70% is carried in vessels of less than 5 000 tons or, say, 7 500 tons dead weight, and on a draft of less than 25 ft. If a channel 30 ft. deep is provided, it would be available to nearly 90% of the vessels of the world. The figures can be verified by reference to "Lloyd's Register".

A great deal of comment and objection has been made to the fact that even with a 25-ft. channel, the speed of a vessel would be considerably reduced. Others have made it appear that at least 5 to 6 ft. of water would be required below the keel of a vessel to maintain a speed of 12 miles per hour, and from this they argue that a draft of only 19 to 20 ft. could be used on a 25-ft. channel. Although it is true that, in restricted channels and shallow water, the speed would be reduced, even to, say, 5 miles per hour, the percentage of time so lost as compared with the whole voyage is insignificant. What again is the experience of vessels operating in the open seas? It is common knowledge that through stress of weather, fogs, etc., vessels are often delayed one or two days or even longer on a given voyage, and yet vessel owners do not give up their business because of this fact. It is absurd therefore to state that a vessel requires 5 or 6 ft. below her keel to maintain full speed through channels, even if this were possible from other consideration as to the canal banks, etc.; and, moreover, actual experience with ships shows that owners will load their vessels to the maximum safe limit of draft, and take the small delay due to reduced speed. The profit due to the extra cargo thus carried would seem to be evident, even to a layman of ordinary intelligence. On the Great Lakes the vessels operate through the shallower channels, not with some feet of clearance, but more often with inches, and do so successfully.

It is absolute nonsense therefore to make a mountain out of the molehill of reduced speed over an insignificant part of a voyage, particularly when one considers that the average speed of a freight train is about 4 miles per hour, or less, when terminal and other delays are taken into account; and yet the speaker ventures to suggest that, in proposing a new railroad, these delays would not deter its construction.

Another objection has been raised to the proposed waterway, in that it would be open for only about 7½ months in the year, and, therefore, that the business would be seasonal. With the exception of the trade by regular liners to the larger ports of the world, all the general ocean trade is seasonal. A prominent ship-owner once told the speaker that operating a freight vessel itself did not require much effort, but the really successful ship-owner was the man who knew the geography of the world and where cargoes were to be had at different times of the year. One need only call attention also to the Baltic trade where somewhat similar conditions pertain.

Cargo vessels to-day are navigating the whole globe, and often go long distances, even in ballast, to pick up a cargo. To argue that a vessel would not come up into the Great Lakes because it is so far, is laughable, and to state that: "Vessels will not find it profitable to gather up partial cargoes", and

"such attempted service would be entirely unprofitable and uneconomic", shows a pathetic ignorance of facts. When one considers the long ocean voyages that vessels make, without any port of call, it is inevitable that the great area of the Middle West economically adjacent to the Great Lakes, with its numerous ports, and with a tremendous producing and absorbing power, would prove an attractive and profitable field for the cheap form of transportation offered by ships. Again, the speaker would call attention to the successful operation of vessels from the eastern coast of the United States to the Pacific ports, in competition with the much shorter rail haul, to emphasize the fact that actual distance does not act as a deterrent.

Finally, also, even with the restrictions of the present Welland Canal, with its limits of about 250 ft. in length and 14 ft. in draft, Norwegian, Swedish, Danish, Canadian, and other vessels have been operated in the past and are to-day operating through the Great Lakes. This last consideration brings up another argument, of a somewhat specious character, that has been used against the proposed waterway, namely, that ocean-going vessels could not compete with those of the Great Lakes, or that the latter could not operate on the open seas.

All the world over there are special types of vessels designed to do specific work. If changes were made in design so that they would be a compromise between two different sets of conditions, they would probably be failures in both. The lake freighters are the last word in a type of vessel especially developed to do the business of carrying bulk commodities, such as iron ore, coal, etc., in the most economical way, between the different lake ports. Their business is on the lakes, not on the ocean, and experience shows that, even with the bogie of a restricted season, they can be run profitably. To alter the design to allow them to go to sea would be suicidal, and almost as absurd as to suggest putting a passenger vessel of the Hudson River Day Line into the North Atlantic trade, when the summer excursion season was over. No, the function of the ocean-going vessel would be to carry commodities to and from the Great Lakes and the remainder of the ports of the world, including even those on the east and west coasts of the United States. She would not, and, if a foreign vessel, she could not, according to the coastwise shipping law, engage in inter-lake commerce; but lest the opponents' arguments should be taken too seriously, the speaker will quote two which are destined to appeal to the sense of humor of any one connected with ships. The first is as follows:

"Lake vessels with their flat bottoms have a capacity of 14 000 tons, but ocean-going vessels with different keels would be limited to a capacity of not more than 4 000 tons, * * * ."

The shape of the mid-ship section and the bottom is identical in both lake and ocean cargo vessels, and although some of the former may be of a little fuller form at the ends, there is absolutely no difference for the middle body. Again, in criticizing a statement in McElwee and Ritter as to the cost of a large lake vessel, an enthusiast makes bold to say* that "the present cost of such a vessel is about $400 000, or a little over one-third of the amount stated [$1 100 000]". At one time, before the World War, a vessel of 10 000 tons was

* Twelfth Annual Rept., New York State Waterways Assoc., 1921, p. 44.

built for about the sum quoted, but not since then, as any one can find out by inquiring of any lake shipbuilder. Consequently, the critic's statement that the figures quoted in the book are "utterly unworthy of trust", should be politely returned to him with the suggestion that it is always the part of wisdom to make sure of one's own facts before contradicting others that are true.

A final argument which is somewhat related to the foregoing, is that ships of a peculiar type would have to be specially designed for this service. This is nonsense because any vessel within the limit of draft available could use the Welland Canal and the great open waterways of the Great Lakes, in precisely the same way as it does at present all over the world. One does not hear of specially designed vessels to navigate the Delaware River, or the River Clyde, or the Suez or Manchester Ship Canal waterways where ships take the time and risk of restricted navigation in order to reach the final port. It is nevertheless true that vessels for some particular class of work might require special designs, in order to obtain the maximum efficiency for that special service. If so, the problem presents no difficulty to the naval architect or ship-builder, and is nothing more than he is continually being called on to solve. Most of the coasting types and lake package freighters could be used for coastwise service on this route without any modification. Perhaps one might interject the thought here that, after all, large numbers of the trans-Atlantic liners are very specialized types for that particular trade and, as such, do not represent the trading vessels of the world.

The following general conclusions may be drawn, not necessarily from masses of minute computations, or timid predictions as to the future, but from a common-sense consideration of what vessels are actually doing to-day all over the world, and from past experience of developments due to increased water-transportation facilities:

1.—There is nothing to hinder existing sea-going vessels up to about 7 500 tons dead weight, or even a little higher, from utilizing the proposed St. Lawrence route to reach the great productive Middle West.

2.—Delays from locks, fogs, ice, and reduced speed in shallow waters, representing only a small percentage of the time of a voyage of an overseas vessel, are difficulties that have fallen to the lot of the ship-owner from time immemorial and form no serious obstacle. Fog particularly is rapidly losing its menace with recent developments in submarine signaling.

3.—Vessels will always go where trade is to be had, irrespective of the fact that it may take a little longer to reach a certain destination; and the Middle West, with its numerous lake ports, has already shown its productive and absorbing power, as reference to trade statistics will show.

4.—If special types of vessels for the intercoastal trade should be desired for any specific purpose, their design would present no difficulties. In this connection, it may be remarked that the actual coast line of the United States and Canada available for coastwise shipping will be increased thousands of miles by opening up the Great Lakes to the Atlantic Ocean. This alone has enormous possibilities for increased trade.

5.—It is absurd to talk about competition between special lake types and ocean-going or overseas vessels, as each performs an entirely different kind of service.

In conclusion, and referring again to past experience, it is the man with the broad vision, undimmed by the immediate existing order of things but looking to the future, who has been responsible for the progress of the world.

The United States and Canada are new countries still possessing tremendous possibilities for future development both in material and population. Cheap transportation is the essence of trade, and the speaker ventures to suggest that, in the years to come, the proposed Great Lakes-St. Lawrence Waterway, the New York State Barge and Erie Canals, and the Mississippi River, will not only prove inadequate for the tasks imposed on them, but that even other waterways already suggested will be necessary for the economic life of the two countries; and, then as now, the two great banking centers of Montreal and New York will still continue to feel an enhanced benefit from a prosperous and busy Middle West, even though they may lose a few dollars in tolls on shipping.

F. P. WILLIAMS,* M. AM. SOC. C. E.—In this paper there is presented an extended description of the St. Lawrence navigation project, the cost of which it is proposed to pay from power developed along the Niagara and St. Lawrence Rivers.

Much convenient data regarding present conditions are furnished. The description of present conditions along the route from Duluth to the sea is of special interest. Many phases of the subject are discussed, with references extending back twenty-five or thirty years, showing the author's familiarity with the subject.

Although the author (page 1261†), refers to the control works on the Niagara River for the dual purpose of maintaining the best available depths, and, at the same time, of creating storage for water power in the Niagara and St. Lawrence Rivers, he does not make it clear in this or in the varied references to navigation and water power intermixed throughout the paper, that the principle of regulation shall conform to the requirements of navigation at all times, and that the development of water power shall be subordinate to navigation requirements. The accepted principle is that water power shall be subordinate to navigation.

In the following discussion the speaker desires to consider various points under their respective headings.

The Subsidizing of Navigation by Water Power.—In the paper it is stated, in numerous places, that water power should subsidize navigation. For instance:

"* * * that the construction of this waterway is inevitable, * * *, and that a demonstration of its commercial feasibility is unnecessary." (Page 1268.†)

* * *

* Secy. Engr., New York Water Power Comm., Albany, N. Y.

† *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions.

"It is of little concern whether the Rapid Sections of the St. Lawrence Waterway to the Sea has in the first decade a commerce of 5 000 000 or 20 000 000 tons." (Page 1268.*)

Again (page 1271.*):

"* * *. The undertaking is, therefore, a commercial project and the United States does not need to contribute toward its cost."

The advantages accruing to navigation by using power to subsidize it are gained at a corresponding sacrifice of the rightful advantages belonging to power. Both navigation and power are important necessities. There has been a great deal of looking forward to advantages to be obtained from large quantities of power commercially developed on the St. Lawrence River. No general advantage is obtained by helping navigation at the expense of power.

With a liberal supply of power ranking high as an essential of economic welfare and modern comfort, and with navigation a prime necessity, it is well to consider the benefits derived from each, and also to consider financing that will not sacrifice the benefits to be derived from power to the cause of navigation, important as navigation is. The subsidizing of navigation by water power, as advocated by the author, is opposed to this and is open to serious criticism on that account.

Comparison of Waterway Improvement with Cost of Power Development.—It is important to note the trend in plans for the improvement as shown by a comparison of the Government Deep Waterway plan for navigation, made in 1900 and revised in 1918, with the Wooten-Bowden plans submitted to the International Joint Commission in 1922. In the first plans, navigation alone was considered. The scheme was to improve the International Section of the river by a series of dams and locks, and to construct a channel so as to reduce the velocities to conform with the requirements of navigation. It was not planned to extend the dams across the river, forming power dams, and thus change conditions by flooding. The total cost of such plans, revised to 25-ft. draft, in 1918, was estimated at $33 962 772. This estimate is doubtless low on the basis of current prices.

The plans of the engineers reporting to the International Joint Commission in 1922 assume the construction of extensive dams that will raise and change the river elevation extensively, and create power possibilities of almost 2 000 000 h.p. Such plans are natural in view of the trend in the power industry. The tremendous growth in the demand for electric power and the improvements in transmission make it feasible to develop the St. Lawrence power at this time, whereas, at the time of the early studies, such development was out of the question. The cost of the International Section, according to the engineers' estimate, is $201 850 583, of which $135 047 264 is chargeable to power. Thus, it is seen that a navigation project, with navigation alone in view, may be had for about one-fifth of the present-day estimate for the combined project. Incidentally, in the present-day combined project, two-thirds of the cost is chargeable to power,

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions.

No one desires for this river waterway improvements that cannot be adapted to the full utilization of power. Similarly, no one desires water-power improvements that do not fully conform to navigation requirements. The point worth considering is the relative amounts of moneys required to secure power and to secure navigation. This unique proposition as offered urges undertaking a navigation project and combining with it a power project costing from four to five times the amount of the navigation project alone and, incidentally, making power pay for the entire improvement.

New York State's Interest in Waterway.—In recent years, New York State has paid more than $160 000 000, without outside aid, to connect the St. Lawrence River and the Atlantic Ocean, for the purpose of lowering rates and for carrying American shipments through American ports. The State owns one-half the bed of the St. Lawrence River along its border, and is the natural agency to develop and control the power on the St. Lawrence.

New York State will be required to pay a large percentage of the cost of the proposed project, in the ordinary procedure of paying her share of Government expenditures. The plan proposed by the author amounts to making New York State alone pay the entire cost of the navigation project in so far as the Federal Government's share of the enterprise is concerned. This to be brought about through increased charges for power due to the increased cost placed on power developments to cover navigation improvements. It is one thing to contemplate a great enterprise such as this, when the advantages of the main objective—the waterway—stand out as warranting the investment; it is another thing to contemplate this enterprise in so far as New York State's interests are concerned, as appraising the Barge Canal as an "escape" from the Niagara to the Hudson River,* and as giving New York interests in the water power along the Niagara and St. Lawrence Rivers over to the benefiting of navigation interests, without regard to whether or not the improvements in the waterway are warranted.

Chicago Diversion.—Under the heading "Water Power"†, the author assumes that the diversion of 10 000 cu. ft. per sec. at Chicago will continue for a number of years. No reference is made to the fact that a case is pending in the United States Supreme Court between the Government and the Sanitary District of Chicago to enjoin the Sanitary District from diverting more than 4 167 cu. ft. per sec. from Lake Michigan.

Subsequent to the presentation of Mr. Shenehon's paper in October, 1924, the Supreme Court granted to the United States an injunction restraining the Sanitary District of Chicago from withdrawing more than 4 167 cu. ft. per sec. from Lake Michigan after March 5, 1925. A revocable permit was issued by the Secretary of War dated March 3, 1925, based on the recommendations of the Chief of Engineers. The significant part of the permit relating to the author's assumption of a continued diversion of 10 000 cu. ft. per sec. from Lake Michigan at Chicago, is as follows:

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1262.
† *Loc. cit.*, p. 1274.

"* * * to divert from Lake Michigan * * * an amount not to exceed an annual average of 8 500 cubic feet per second * * *."

* * * *

"That the Sanitary District of Chicago shall carry out a program of sewage treatment by artificial processes which will provide the equivalent of the complete (100%) treatment of the sewage of a human population of at least 1 200 000 before the expiration of the permit.

"That the Sanitary District shall submit for the approval of the Chief of Engineers and the Secretary of War plans for controlling works to prevent the discharge of the Chicago River into Lake Michigan in times of heavy storms. These works shall be constructed, in accordance with the approved plans and shall be completed and ready for operation by July 1, 1929.

"That if, within six months after the issuance of this permit, the City of Chicago does not adopt a program for metering at least 90% of its water service and provide for the execution of said program at the average rate of 10% per annum, thereafter, this permit may be revoked without notice.

"That if, in the judgment of the Chief of Engineers and the Secretary of War, sufficient progress has not been made by the end of each calendar year in the program of sewage treatment prescribed herein so as to insure full compliance with the provisions of Condition 4, this permit may be revoked without notice.

"That this permit, if not previously revoked or specifically extended, shall cease and be null and void on December 31, 1929."

The significant part of the recommendations of the Chief of Engineers relative to the assumption by the author of the continued diversion of 10 000 cu. ft. per sec. from Lake Michigan at Chicago, is as follows:

"7. * * *. (The execution of sewage treatment program which will relieve the load on the Drainage Canal by the equivalent of a population of 1 200 000 * * * will make possible a reduction in the amount of diversion to 7 250 cu. ft. per sec., or lower, by the end of 1929). This condition looks to a reduction to 4 167 cu. ft. per sec. by 1925."

* * * *

"12. * * * Condition (3) (The construction of controlling works to prevent the Chicago River from discharging into Lake Michigan in times of heavy storms * * * should be ready for operation July 1, 1929), is considered necessary to permit an ultimate reduction of the diversion to 4 167 cu. ft. per sec."

Engineers are left to their own conclusions regarding the appropriateness of such an assumption by the author, that the diversion of 10 000 cu. ft. per sec. at Chicago will continue for a number of years.

Conservation of Coal Supplies.—Under the heading, "Value of Added Water",* the author states that there is no vital concern in regard to the saving of bituminous coal, because at the present rate of consumption the visible supply in the United States will last for more than 5 000 years.

The following is quoted from a *Bulletin* dated August 1, 1922, issued by G. H. Ashley, State Geologist of Pennsylvania:

"Bituminous Coal Reserves in Pennsylvania, in Short Tons, 1922.

"Original deposit	75 259 055 000
Mined out	5 519 665 000
Recoverable	43 830 860 000"

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1285.

If the mining of bituminous coal at the rate of 170 000 000 tons per year is continued, the bituminous coal in Pennsylvania should last about 250 years. For years the State of Pennsylvania has supplied about one-half the coal used in the United States. It is true that the coal originally estimated to be underground in Pennsylvania constitutes only a very small percentage of that in the whole United States, but the point is that (1) the State of Pennsylvania is depended on to-day to furnish a large quantity of coal because it exists there in thick veins; and that (2) the cost of coal is mounting. There is only one conclusion, that while estimates may show a supply of coal sufficient to last a long term of years, there should be vital concern about how long the supply of cheap coal will continue. All water power should be developed as early as it is economical to do so, for the purpose of conserving the coal supplies.

Radius of Economic Benefits from Water Power and from Navigation.—Under the heading, "Water Power and Navigation",* the author states that while the economic benefits of water power may be limited to 200 miles, navigation may benefit an area in a radius of 2 000 miles. It is difficult to assign specific distances, as the author has, to either range of benefits when the indirect and intricate benefits involved in manufacturing aided by navigation, by power, or by both, are considered.

The manufacture of some of the electro-chemicals, paper, etc., is only feasible when large quantities of cheap hydro-electric power are available. In such instances, instead of limiting the benefits from hydro-electric power used in such manufacture to 200 miles, it is more nearly correct to estimate that the benefits are country-wide. It is possible that the author has in mind simply the feasible distance that power may be transmitted. Plans advanced for disposing of St. Lawrence power contemplate transmitting it 300 miles to New York and elsewhere.

Summary.—Only a part of the paper is here discussed, the speaker's remarks being made to give students of the project information from which they may see what the situation is, namely, that the author is advancing a plan to nationalize the water-power interests of the State of New York along the Niagara and St. Lawrence Rivers, making the power subsidize navigation improvements, without even taking the trouble to justify such action from either the legal or economic standpoint. The speaker heartily disapproves of this plan.

The author has spoken of opposition as stimulating. It is hoped that as a result of this suggestion, a discussion of all phases of the points touched upon regarding this important subject will be had.

THOMAS H. HOGG,† M. AM. SOC. C. E.—The author is to be congratulated on the excellent manner in which he has presented his description of the projected waterway to connect the Great Lakes and the sea.

The speaker will endeavor to point out certain fallacious reasoning that has led to serious erroneous conclusions on the part of the author. He assumes that water power should be charged with the cost of navigation development

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1274.

† Chf. Hydr. Engr., Hydro-Electric Power Comm. of Ontario, Toronto, Ont., Canada.

on the St. Lawrence River. There are two possible conditions under which a justification of this assumption might be considered:

1.—That the development of water power causes existing navigation structures and channels to become inoperative.

2.—That the benefits derived from the power developed are shared by districts and interests identical with those benefited by the navigation works.

Present conditions on the St. Lawrence River provide for navigation with a draft of 14 ft., whereas the author's proposal is for deep-draft navigation. It would be reasonable, under certain circumstances perhaps, for power developments on the river, if they destroyed existing works, to be charged with facilities equivalent to those existing, but it is quite unreasonable to expect a power development to go beyond the existing facilities and pay the whole of the enormous increased cost occasioned by the construction of greatly improved river channels, canals, and locks suitable for vessels six or eight times as large as those now accommodated by the existing canals.

At present, power is transmitted to points about 250 miles distant from generating stations, so that potential developments are of possible value throughout a circle described by this radius. This transmission distance may, perhaps, be somewhat extended in years to come, but at present, and to a greater or less extent in the future, it gives a measure of the district benefited by a water-power development, and also a measure of the district which must pay through power rates for the development and for all the works provided for in the construction cost of such development.

On the other hand, navigation benefits in this particular instance extend far beyond this radius of 250 miles, and, in fact, reach their utmost value in a district greatly removed from the locality of the power development. The districts that would be especially benefited by the construction of deep-draft navigation works on the St. Lawrence River are the eastern section of the Prairie Provinces in Canada and the Central United States.

The conclusion to be reached, therefore, is that, according to the author's assumption, industry and other power users in the district bounded by Toronto, North Bay, Quebec, Boston, and Rochester, should alone pay all the cost of these navigation works, which would benefit primarily the district west and southwest of Detroit.

Engineers and financiers are quite conversant with the economic aspects of the construction of water-power plants and of the somewhat temporary nature of the market and employment involved therein. The relation of the water-power development to the permanent market for raw materials, production of manufactured goods, and development of permanent employment in other industries, is not so well appreciated and, in fact, by many, is not appreciated at all.

An analysis of these general aspects of the problem under consideration, aided by a special census in the United States, shows that of the whole investment in a water power and the industries built up by it, only 13.4% is found in the water-power development, and 86.6% in the applications of the power. In the matter of employees, less than 3% of these are engaged in the pro-

duction of power, and more than 97% in industries utilizing that power. The wages paid are 2.3% of the total for water power, and 97.7% in application of this power.

Very great and widespread benefits are received by all people and interests in the district affected by the production of power through the application of that power to industry; but the benefit derived from the power *per se* is usually small, accounting as it does in its tolls for less than 3% of the business of the district. Thus, power development is in the same category as navigation works, and instead of being forced to pay tribute, water power might more fairly be subsidized.

The author indicates that by regulation works constructed at the outlets to Lake Huron, Lake Erie, and Lake Ontario, in addition to those existing on the St. Marys River, a dependable flow of 230 000 cu. ft. per sec. can be obtained for the St. Lawrence River on the assumption that 10 000 cu. ft. per sec. now diverted at Chicago passes to the sea by its former and natural course. The argument appears to be—although not so stated—that, since the power capacity on the Niagara River and on the St. Lawrence River is increased by this regulation of the Upper Lakes, the people in districts to be served by these power developments should pay the whole cost of the necessary regulating works which are so vital for navigation.

A comparison is made by the author of the results to be obtained by regulation of the St. Lawrence River, according to the Government engineers' report on the St. Lawrence Waterway, and the results, he asserts, can be obtained by complete regulation of the Upper Lakes. It must be kept in mind that, in the various studies of regulation already made, different objectives are set. In some cases this objective has been the maintenance of a large flow in the Lower St. Lawrence River during September, October, and November; in others, the maintenance of proper winter operating conditions; or, again, the maintenance of a high uniform flow. In passing, it may be noted that it is quite possible to obtain a higher minimum flow by the regulation of Lake Ontario alone than the author quotes from the Commission engineers' report, for purposes of comparison with his figure of 230 000 cu. ft. per sec. It must always be kept in mind that, although artificial regulation, even after a subtraction for the Chicago diversion, may give better results than those obtained under natural conditions, yet the results would be just so much better were there no diversion at all at Chicago. In other words, the diversion at Chicago and the regulation of the lakes are two perfectly independent matters.

There are certain other points on which exception might also be taken, for example, the author's assumption that there would be no break in cargo at Montreal after the construction of the deep waterways. It is conceivable that economic operation will involve the use of large lake freighters from the Upper Lakes to Montreal and transshipment there to sea-going freighters.

One must, of course, at this time speak with caution respecting the diversion of water at Chicago, for although this is a most important international matter, yet at the present time its domestic aspect, as far as it concerns the

United States, is under consideration by the Supreme Court. It may be that the author's conclusion that for the next fifty years, that diversion will be at least 10 000 cu. ft. per sec., may not be borne out by that decision.

Although several relatively minor causes of lowered lake levels are noted by the author, nevertheless proper emphasis has not been placed on the level-lowering effect of the Chicago Drainage Canal, which removes water not only from the navigation channels, but entirely from the water-shed, thereby lowering all lake levels except that of Lake Superior and all river levels from that lake to the sea. In this connection it is not practicable to compensate the St. Lawrence River for its loss.

It is to be hoped that the author's interest as one of the engineers in the employ of the Chicago Drainage Canal has not caused him to place undue stress on the necessity for navigation and power interests to subsidize the City of Chicago in its diversion. This possibly may be the reason he has assumed such an unsound economical basis as that of placing on power developments an undue share of cost for the protection of navigation works and lake-level regulation without regard to whether power development had been a primary cause of the great lowering of lake levels or whether such development has been a prime factor in necessitating deeper and enlarged waterways.

The author's apparent concern in regard to the failure of power development companies to expend money for ice protection at the outlet of Lake Erie is probably due to the fact that he has failed to recognize, in the present instance, that power development engineers are unable to spend millions of dollars for accessory works that cannot be made to return a justifiable income on the money invested.

L. C. Sabin,* M. Am. Soc. C. E.—It is refreshing to have such a comprehensive paper devoted to the Great Lakes Basin and the water power possibilities of its final outlet to the sea presented by one who has given much time and thought to the collection of data concerning it, and to the subsequent study and application of these data.

The author is correct in his assumption that the justification of a deep waterway outlet from the Great Lakes to the sea must be found, from the point of view of navigation, in the length of ship channels already available and intensively used in the Great Lakes themselves which would be tapped by such a deep-water outlet. Duluth, at the head of Lake Superior, 1 000 miles by steamer from Buffalo, is only 200 miles nearer, by airplane, to the east coast of Maine than to Seattle. As the crow flies, Chicago is equi-distant from Denver, New Orleans, and Quebec.

The advisability of building the St. Lawrence waterway at this time has been the subject of much discussion. The promoters of the project doubtless have grossly overestimated, at least by inference, the amount of traffic which it would serve in the immediate future, and the resulting saving in transportation costs. The speaker has seen no well considered study of this economic

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question, and from the standpoint of navigation it is a study that should precede, or at least coincide with, the study of the physical facts.

If the St. Lawrence improvement were for the sole purpose of navigation, it is doubtful whether its cost could be justified, even with the advantage of its connection with lanes of commerce already established over which traffic is hauled at a cost hardly approached by any other system of transportation except on the ocean. Fortunately, however, it is not merely, and perhaps not mainly, a navigation matter. The value of the water power is such that whenever the large blocks of power developable can be advantageously sold, the returns should pay interest on the entire cost, and, in a term of years, amortize the investment as well.

While those in favor of the project may have painted too glowing a picture of the probable volume of commerce and the consequent benefits to be derived from cheap carriage, the opponents, on the other hand, have attacked the estimated cost on the ground that the improvement of the Great Lakes channels must be a necessary part of the plan, and that the cost of such improvements should be included in the estimate.

It is well known that a large part of salt-water commerce is carried on drafts of 25 ft. or less. The tentative plan for the St. Lawrence provides for this depth at present, with a possible increase to 30 ft. later should this become desirable. The same idea is applied in the new Welland Canal. It may be assumed that a navigable depth in excess of 25 ft. in the improved St. Lawrence will not be undertaken until ocean-going tonnage has so increased as to make it an economic measure. Accordingly, present consideration may be confined to the justification for a navigable depth of 25 ft. on the present commerce lanes of the Great Lakes as well as on the St. Lawrence outlet.

The importance of the commerce on the Great Lakes is not generally appreciated. In volume it equals the foreign commerce of all ports on the Atlantic, Gulf, and Pacific coasts. It exceeds the freight carried on 297 rivers, canals, and navigable waterways of the United States, including 37 well-known streams, such as the Ohio and Mississippi Rivers, and their tributaries above New Orleans, the Hudson River above New York, the Delaware River above Philadelphia, etc. These figures exclude that part of the river traffic attributable to ocean ports, such as New York, New Orleans, Philadelphia, etc. More tonnage passes the Sault canals annually than the combined traffic of the Erie, Manchester, Kiel, Suez, and Panama Canals. The average eastbound cargo at the Sault canals in 1923 was nearly 7 000 tons, while the average cargo on the Panama Canal was about 5 600 tons.

Not only in volume, but particularly in length of haul, do the Great Lakes channels exceed other waterways, exclusive of ocean tracks. The ton-mileage of the Great Lakes is about sixteen times that of the thirty-seven waterways mentioned. For the year 1923, it was equal to about one-sixth the ton-mileage of all the railroads in the United States.

The author quotes* from the Presidential Address of the late Alfred Noble, Past-President, Am. Soc. C. E., of 1903, in which, on the basis of the actual

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1244.

freight rate on the lakes of 0.89 mill and estimated rail rate of 3 mills per ton-mile, the saving in freight charges for the single year, 1902, was estimated at $63 000 000. The question may be raised as to how this saving is distributed, and it may even be thoughtlessly suggested that the lake transportation interests are the principal beneficiaries. A moment's consideration, however, will show that it is the same kind of a benefit as that resulting from a time-saving improvement in the making of steel and the grinding of flour, and that the beneficiaries are those who buy the ultimate products, whether wash-boilers or steel bridges, brown bread or Parker House rolls.

In the twenty-one years that have elapsed since 1902, the year of Mr. Noble's comparison, that part of the Great Lakes commerce that passed to and from Lake Superior has amounted to 1 337 000 000 tons, and the freight charges to $1 000 000 000, with an average rate of 0.9 mill per ton-mile. Using the figure of 3 mills per ton-mile as comparative rail cost, the saving in freight charges amounts to $2 300 000 000. Subtracting from this the cost of all channel and harbor improvements on the entire Great Lakes system, there remains a net saving of $2 000 000 000, the interest on which at 3% is $60 000 000 per year. With such a surplus to the credit of Great Lakes channels no hesitation need be felt in advocating further expenditures whenever the needs of the growing commerce require more adequate facilities.

The author mentions the value per inch of draft as $500 000 for one year. This figure is not a guess but an estimate based on the actual number of passages of loaded vessels of the larger classes during 1923, which showed an increase in carrying capacity of more than 3 000 000 tons for an increase in draft of 6 in. At the average freight rate of 88 cents per ton this would amount to very nearly $3 000 000 extra earning capacity, reduced only by a small additional cost for loading and unloading and a trifling longer delay in port.

About 30% of the freight traffic is carried by vessels capable of drawing from 23 to 24 ft., and a draft of 21 ft. could be used by vessels carrying 95% of the freight. With the present fleet it is estimated that a total gain of $15 000 000 per year would result were a 25-ft. channel made available.

To sum up the case for Great Lakes navigation, the savings by this system have piled up a surplus of $2 000 000 000 in the last two decades alone. The savings amount now to $100 000 000 per year. A further saving of $15 000 000 per year awaits the development of deeper channels.

Two fundamental methods present themselves by which deeper channels may be obtained. It is a physical possibility to raise all the lakes by 4 or 5 ft. With comprehensive regulation, meaning thereby control of river slopes as well as of lake levels, the deeper channels would result. If this were attempted and the St. Lawrence outflow entirely cut off, it would take about 20 months to accomplish the initial rise. If the St. Lawrence flow were reduced 20%, it would take 8 years. This plan, however, is manifestly impracticable as resulting land damages would be too great, and the value of existing port facilities would be almost entirely sacrificed.

The second method is the excavation of channels to the required depth wherever this depth does not naturally exist. The author suggests a combination of these two methods, which may be referred to as a third plan. The advisability of such a combination will hardly be disputed, although the question as to the extent to which one or the other fundamental method should be applied is one on which opinions will differ, and which merits discussion.

It will be well first to get a clear understanding of some physical facts concerning the Great Lakes—facts which are not always correctly stated and used in discussions of the hydraulics and hydrology of this great reservoir system. The fact that the water levels of the Great Lakes are now comparatively low, especially on Michigan-Huron with its thousands of miles of shore line, has attracted wide attention, and has led to much speculation regarding the cause, particularly among laymen. Having available definite gauge readings on each lake extending back to 1860, the extent of this lowering may be determined, and it appears that the average level for 30 years (1890-1919), is below the level of the preceding 30 years (1860-1889), by the following amounts for the several lakes: Superior, 1 in.; Michigan-Huron, 17 in.; Erie, 9½ in.; and Ontario, 8 in. For the last four years the supply has been unusually deficient; in fact, on some of the lakes deficient to an unprecedented degree for so long a period. Compared with the 60-year mean (1860-1919), the average level for 4 years (1920-1923), has been low by 2½ in. on Lake Superior, 15 in. on Lake Michigan-Huron, 7½ in. on Lake Erie, and nearly 9 in. on Lake Ontario.

Based on the discharge values of the St. Lawrence River given in the 1921 Report of the United States and Canadian Government engineers on the St. Lawrence improvement for the years, 1860-1887, and the discharge values as determined by the U. S. Lake Survey for subsequent years, it appears that the supply to the water surface of all the lakes for the 30 years, 1890-1919, was 5.5 ft. less than for the 30 years, 1860-1889; or, in other words, that the average shortage in supply for the later 30 years was about 2¼ in. per year. This represents a decrease in supply of about 6 per cent. To indicate the proportion of this deficiency in supply for which Lake Superior is accountable, by comparing the two 30-year periods, the net supply for the later 30 years is found to be lower by 10.9 ft., or the equivalent of 3.6 ft. on the surface of all the lakes. Thus, two-thirds of the total shortage of 5.5 ft. is found on Lake Superior, although the area of its surface is only one-third of the total area of all the lakes. Accordingly, the intensity of shortage on Lake Superior was double that on the other lakes. With this proportion of the lack of supply in recent years attributable to Lake Superior, it is interesting to consider why the average level of Lake Superior was lowered only 1 in. and that of Lake Erie only 9½ in., while the average level of Lake Michigan-Huron was lowered 17 in.

As to Lake Superior, its level was interfered with by various changes in the outlet. The first obstruction took place about 1887 when bridge piers were built near the head of the Rapids and certain small channels were shut off. This occurred near the close of the first 30-year period of record. Other encroachments followed, modifying the equation of flow, and about 1900 the

use of water in side outlets for water power began, but was partly or fully compensated by obstructions. Later, the compensating or regulating works were built, and since 1921 complete control of the outflow has been possible.

By the use of these regulating works, the level of Lake Superior has been held up, the present level being about 1 ft. higher than it would have been had the outlet remained in its original condition with no water used, for power or otherwise, in side canals. It should be noted that this result was accomplished in spite of the fact that the intensity of depleted supply was highest on Lake Superior. Had the restrained water making up this 1 ft. of storage been allowed to discharge gradually in accordance with the original conditions of the outlet, it would have raised the lower lakes a small amount, not, of course, in proportion to the areas, because the discharge of each outlet would have been increased.

A rather definite part of the depression in level of Lake Michigan-Huron in the later 30 years is due to the diversion through the Chicago Drainage Canal; but this diversion affects also Lakes Erie and Ontario. The discrepancy of about 8 in. between the average lowering of Lake Michigan-Huron and that of Lakes Erie and Ontario thus remains to be explained. This apparent discrepancy has sometimes been attributed to scour or to the excavation of navigation channels in the outlet of Lake Michigan-Huron, particularly in the Detroit River. The Board of Engineers on Deep Waterways concluded that scour had taken place at the head of St. Clair River, but comparison of surveys made by the speaker with an old map of 1857 gave no evidence of such a change.

Theoretical calculations indicate that the necessary fall in this connecting channel has not been reduced by channel improvements more than 2 or 3 in. In this connection it should not be overlooked that the improvement of a given reach of channel cannot reduce the fall in the reaches above and below. If in a channel 80 miles long, with a uniform slope and a fall of 8 ft., a reach of 5 miles is so improved as to make it practically a lake, the total fall cannot be reduced more than 6 in. It should also be considered that in dredging operations the material usually is put back in the original river channel, and that it easily may be so placed as to maintain the original slope.

Passing to practical considerations, there exists in the channel connecting Lakes Huron and Erie an enlargement called Lake St. Clair, the level of which, at least in the summer months, is controlled by the levels of the two larger lakes. The level of Lake St. Clair during ice-free months can be expressed by a formula making it a function of the levels of the upper and lower lakes. This formula holds good with residuals of 0.1 ft. or less for 40 years, except for short periods when known obstructions were temporarily in place. There is, therefore, practically no evidence of such a change in the channel as to indicate a reduction of slope between Lake Huron and Lake Erie.

The explanation of this apparent discrepancy in relative water levels of Lake Huron and Lake Erie was investigated by the speaker years ago and the results are shown in his report on the discharge of the St. Clair River

made in 1902.* During the winter months the outflow from Lake Michigan-Huron is reduced by the action of ice in the St. Clair and Detroit Rivers. An ice cover usually forms over a considerable portion of this connecting channel, which reduces the discharge according to the laws of flow, but a much greater effect results from ice jams that occur at times, especially in the St. Clair River.

This ice effect was evaluated by discharge measurements taken from January to April, 1901, which showed individual determinations as low as 75 000 cu. ft. per sec., or considerably less than one-half the normal discharge. Connecting these discharge measurements with the readings of gauges established in 1899 at the head of the river resulted in an equation for winter discharge from which it appeared that for the three winters, 1901-03, there was an average reduction in flow of 50 000 cu. ft. per sec. for 4 months of each winter, equivalent to about 0.41 ft. per year on the surface of Lake Michigan-Huron. It is apparent that variations in this ice effect are sufficient to account for temporary changes in the relative levels of Lake Michigan-Huron and Lake Erie.

The gauges at the head of the St. Clair River have been maintained since 1900, but there is no reliable method of determining the winter discharge prior to that date. The effect of ice varies from year to year according to the severity of the winter, not following closely the mean temperature, but probably dependent on the number of short periods of extreme cold followed by temporary break-ups.

The speaker's explanation of this discrepancy in the levels of Lake Huron and Lake Erie is that the average ice effect in the earlier 30 years of record, which cannot now be evaluated, was greater than that of the later 30 years. This cannot be definitely proved, but some light can be thrown on it by a consideration of the indicated local supply to Lake Michigan-Huron for 60 years. The local supply to any lake is determined by adding the discharge to the storage and subtracting the inflow. The inflow to Lake Michigan-Huron from Lake Superior is known in winter nearly as well as in summer; but in determining the outflow from Lake Michigan-Huron prior to 1900 the figures for discharge for the ice period are generally too high, and accordingly give too high a local supply for the winter months.

A comparison of the net local supply to Lake Michigan-Huron for the 40 years, 1860-1899, with that for the 20 years, 1900-1919, shows the indicated net supply for the former period to average 3.036 ft. per year and for the later 20-year period, 2.816 ft., indicating a shortage in net supply of 0.22 ft. per year.

This retardation by ice, however, occurs entirely in the half year from December to May, inclusive. Dividing the yearly supply into two parts, the period from December to May being called the first half year and that from June to November the second half year, it appears that for the earlier period of 40 years the net local supply to Lake Michigan-Huron averaged 2.071 ft. in the first half and 0.965 ft. for the second half. Of the later 20-year period,

* Report, Chf. of Engrs., U. S. Army, 1902, p. 2825.

1900 to 1919, inclusive, there is a fair knowledge of the winter discharge. This shows the net local supply to average 1.712 ft. per year in the first half and 1.104 ft. per year in the second half. Therefore, while the average supply for the second half of each year during the later 20-year period actually exceeds that of the corresponding half of the year for the earlier period by 0.14 ft., the average supply for the first half of the year for the later 20 years is 0.36 ft. less than the indicated supply for the corresponding part of the year in the earlier period.

It is submitted that such a condition is not likely to have occurred and that the figures for the winter discharge of the St. Clair River prior to 1900 are probably too high by from 0.36 ft. to 0.50 ft. The result for 3 years (1900-1902) indicated a reduction of flow equivalent to 0.41 ft., as already mentioned. These results are mutually confirmative. Correcting the years prior to 1900 by 0.41 ft. each, shows the average net supply to Lake Michigan-Huron for the 30 years, 1860-1889, to be 2.74 ft. on the lake surface and for the 30 years, 1890-1919, 2.64 ft., a difference of but 0.1 ft. per year, or less than 4 per cent. Reduced to the area of all the lakes this represents a shortage of only 0.05 ft. per year, or about 1½ per cent.

Briefly then by comparing the record of 30 years (1860-1889) with that of the following 30 years up to 1919 there appears a total shortage in supply to all the lakes of 5.5 ft. in 30 years, an average reduction of 0.18 ft. per year, or 6 per cent. Of this shortage of 5.5 ft. Lake Superior's supply deficiency accounts for 3.6 ft., or 65%, Lake Michigan-Huron's deficiency for 1.5 ft., or 27%, leaving only 0.4 ft., or 8%, to be allocated to Lake Erie and Lake Ontario.

The author, using perhaps different data as to the outflow from Lake Ontario, finds a smaller deficiency in supply, only about 3%; but taking the larger figure of 6%, or 5.5 ft., in 30 years, an average of 2¼ in. per year, is this a sufficient change to warrant the conclusion that the trend is definitely downward, and to launch a discussion of changes in climate, permanently reduced precipitation, and the effect of clearing the forests? The speaker thinks not. On the other hand, he believes that the experience of the past, and particularly of the last three or four years, should not be allowed to pass without profiting by the lesson that natural water levels recede and that to whatever extent they may be artificially sustained, datum planes for improvement should be set low enough to assure continuous maintenance of the navigation depths found to be economically justified. This depth at the present time is considered to be that permitting a draft of not less than 24 ft., which the largest lake carriers are now able to utilize and for which many more would be built within the ten or fifteen years required to make the improvement.

In considering to what extent lake-level regulation may replace the improvement of channels by dredging, it is desirable to state some principles that appear axiomatic regarding the operation of reservoirs. Two of these were well stated by the late H. M. Chittenden, M. Am. Soc. C. E., in his paper*

* *Transactions*, Am. Soc. C. E., Vol. XL (1898), pp. 359 and 365.

on the "Reservoir System of the Great Lakes of the St. Lawrence Basin", as follows:

"I.—The effect of a natural reservoir upon a stream passing through it is to reduce variations and irregularities of flow and to give the stream a more uniform discharge upon leaving the reservoir than it had before entering."

* * * * *

"II.—Any reduction of the normal oscillation of level in a natural reservoir will increase the variation of flow in the outlet; and any reduction of the normal variation of flow in the outlet will increase the oscillation of level in the reservoir."

The effect of a uniform lake level is primarily to retain harbor depths. An incidental effect is the stabilizing of the water level for a certain distance up stream from the mouth of the inlet from the reservoir above. The extent of this latter effect depends on the character of the outlet.

Lake Superior, already under regulation, is admirably adapted to this treatment. The rock rim restraining the lake furnishes excellent foundations, and a ¾-mile reach of rock apron below the works insures against scour from concentrated currents. The natural drop over this rock apron was 17 to 21 ft. The level of Lake Superior either under natural or control conditions is free from any effect of the level of Lake Michigan-Huron, the next reservoir below. The section at the regulating works will always remain in control as the lower river will carry any reasonable discharge without affecting the flow through the works which cannot be drowned out to any extent. On the other hand, the slope in the lower river, to accord with present datum planes, will be maintained by the use of the minimum discharge required for power.

With the proposed improvement of the St. Lawrence similar advantages for regulation will be found. The water levels in the river are seriously affected by ice, but under the suggested arrangements the control works cannot be drowned out. When the St. Lawrence is improved for deep navigation these control works will form a part of the plan and the level of Lake Ontario can be regulated. Until the river is so improved, deeper channels below Buffalo are not important.

Turning to Lake Erie, the control of this lake has been the subject of a number of studies, the conclusion usually being favorable with certain limitations. One objection is that if control means a higher average level the occasional storms on Lake Erie would be somewhat more destructive, especially to Buffalo. The principal objection heretofore is that regulation of Lake Erie, without coincident regulation of Lake Ontario, would increase the fluctuations in level of the latter lake, and as concerns the two border countries it happens that the one more interested in the maintenance of levels on one lake is less interested in the maintenance of levels on the other. With the improvement of the St. Lawrence this diversity of interest and its effect would cease.

In the Niagara River the conditions for control are only less favorable than in Lakes Superior and Ontario. Here, again, the ice problem is important, but there need be little fear of the effect of the works being drowned out by the action of the outlet channel below, and if a temporary low controlled discharge depresses the water in the lower river, the effect will not be serious as the navigation there is not important. Accordingly, even if the improvement of the

St. Lawrence should be indefinitely postponed, it is thought that the level of Lake Erie should be controlled, but without a radical increase in its mean level, say, within limits of 1½ ft., or at most, 2½ ft.

As regards the outlet of the largest single reservoir in the system, the St. Clair and Detroit Rivers draining Lake Michigan-Huron, this connecting channel is about 80 miles in length with an average fall of only a little more than 0.1 ft. per mile. The bottom in general is gravel, sand, and clay, except for a rock ledge overlaid with boulders near the lower end. In general, the river is about ½ mile wide and, except at and below the rock ledge mentioned, good navigation depths exist naturally; but where the width is increased to ¾ mile, shoals appear. The regimen, now established by the existing discharge with its relatively small variations, is remarkably stable. With a considerably increased flow capacity the new conditions might be less permanent.

In this lake outlet there is a natural controlling works, or perhaps, more properly, compensating works, set up at times by Nature in the form of retardation of flow by ice. As regards levels, the effect of this natural regulation has in general been good. The trouble is that Nature sometimes neglects to apply the remedy. This natural compensation, if made certain, would be ideal in some respects. It does not seriously affect water levels in the outlet. Whatever effect it has is exercised when the only navigation is in light-draft ferryboats. Thus far, it has not seriously affected any power development. It has to some extent depleted the flow over Niagara Falls and thus affected the grandeur of that spectacle, but Lake Erie fills so rapidly in the spring that by June the flow is restored, and no complaints are known to have resulted from this kind of regulation.

In this paper the mass curve is ingeniously applied to the Great Lakes System to show the maximum value of the continuous yield. By taking advantage of whatever diversification in water-sheds may exist and by the use of the mass curve for mean annual yields, the author is able to show a possible increase of 10 000 to 20 000 cu. ft. per sec. of prime water for power.

It is quite proper that a paper on the St. Lawrence development should be written from a water-power point of view; but to secure this additional volume in actual operation is it not necessary to have the omniscience which is assumed in the method adopted in the paper? Knowing what is to take place in the next year, or the next five years, a decision as to the flow to assume for the earlier period is fairly simple. If any rule has been deduced whereby the several works can be operated without this omniscient mind, it does not appear, and the speaker's experience with similar but simpler problems leads him to doubt the equally smooth working of the theory under conditions of stated limits but of uncertain sequence.

Each one of these lake reservoirs has a considerable fluctuation in local supply. There frequently occur monthly maximum supplies of 1 ft. on Lake Ontario, 0.8 ft. on Lake Erie, 0.6 ft. on Lake Michigan-Huron, and 0.5 ft. on Lake Superior. On the other hand, negative supplies, meaning greater evaporation than local supply from the basin, occur almost every year, the values of these negative supplies being not infrequently 0.2 ft. or 0.3 ft. per month on Lakes Ontario and Erie, and 0.1 ft. on Lakes Michigan-Huron and

Superior. With variations in local supply amounting to 1 ft. per month, the difficulty of maintaining levels within a total range of 1½ ft. may be seen.

No attempt is made in the paper to follow through the details of regulation, the author contenting himself with a broad presentation of the subject. It is of interest, however, to test the possibilities of regulation under the limits laid down. For the condition obtaining in 1910 and 1911, an attempt to restrain the lakes within these limits by the methods suggested by the author would appear to have required changes from the natural flow volumes of the outlets of from 50 000 to 75 000 cu. ft. per sec. in certain months, and still serious oscillation in the level of Lake Michigan-Huron could not have been prevented, but this level would have had a range of 1.75 ft., while in these two years the level would have fallen nearly 1 ft. below the proposed lower limit. This could not have been corrected by increased discharge from Lake Superior, because for these two years the natural rate of discharge from Lake Superior was reduced by regulation 15 000 cu. ft. per sec. below normal, and yet the lake fell 0.3 ft.

In 1924, the level of Lake Michigan-Huron fell 6 in. with an average discharge of only 152 400 cu. ft. per sec. During the four years, 1920-24, the levels of Lakes Superior and Michigan-Huron were depressed about 1.5 ft., with an average discharge of the St. Clair River of approximately 167 000 cu. ft. per sec. Since no excess supply could have been drawn from Lake Superior, this average discharge of the St. Clair River would seem to be as great as would be available for long periods under similar conditions with complete regulation in effect.

It is fully appreciated that the author is merely sketching works and methods, not specifying details, but the speaker is only attempting to point out the possibility of disappointment in the working out of details when this becomes necessary.

With Lake Michigan-Huron under complete regulation, throttling the discharge of the St. Clair River at or near its head will at times result in quite serious depression of level below the works. An indication of the extent of this depression appeared in the plan originally proposed by the author which included locks with lifts of from 1 to 5 ft. As finally submitted, however, (Fig. 5*), the locks have been replaced by navigable passes combined with expansion basins. With Lake Erie at a regulated stage of 573.0 ft., a fall of only about 6.5 ft. would be required to carry a reduced discharge of 160 000 cu. ft. per sec. through the Lake Michigan-Huron outlet, giving a level of 579.5 ft. at the head of the St. Clair River, or about the present low-water datum. Since it is mainly the connecting channels that are shallow the maintenance of levels in the rivers is more effective in reducing dredging costs than the preservation of the levels of the lakes. Under natural conditions high lake levels result in corresponding channel depths, but, as indicated, this condition easily may be disturbed by the character of regulation suggested for such a channel as the St. Clair River.

The proposed plan contemplates the enlargement of channels to take care of excess flow, the latter being required in order to assure the large prime

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1260.

volume of 230 000 cu. ft. per sec. It should be pointed out that in the case of the Lake Michigan-Huron outlet, 80 miles long, it may prove as much of a task to provide this additional flow capacity by dredging, as to provide by the same method the desired depths for navigation throughout the shorter reaches requiring improvement for this prime purpose.

In this connection, the suggestion that the Chicago Drainage Canal contributes an effective share of this needed additional capacity is not tenable unless it be assumed that the flow through that Canal is subject to regulation. To make this clear it is only necessary to assume the flow in the Chicago Drainage Canal to be, say, 175 000 cu. ft. per sec. There would then be, at times, no flow remaining in the St. Clair River to be regulated. Uncontrolled diversions are serviceable in regulation only when the flow is returned to the main outlet or to a lower reservoir of the system, a condition not applying to the Chicago Drainage Canal.

The value for power of the additional prime water which it is proposed to make available by an elaborate system of regulation, would be more impressive if every means were being taken to permit the use of more power at Niagara while preserving the scenic beauty of the Falls. It would also be more impressive if it was certain that all the water power developable on the St. Lawrence could find a ready market, and if, in the plans proposed, consideration were not being given to the entire omission of power works in an important part of that river.

In computations of power benefits the author seems to use a figure of about $7 per h.p. as the annual value. Assuming a final development of 7 000 000 h.p. on the Niagara and St. Lawrence Rivers combined, would give a total annual value of $50 000 000, or less than one-half the benefits now resulting from the Great Lakes navigation. The time may come when the power value of this additional water can be fully realized, but this time has not yet arrived. When it does come, the value will still be below that resulting from navigational uses. Certainly to-day and in the immediate future the needs of navigation should not be sacrificed in the name of power. To require vessels to pass through locks at the foot of Lake Huron or to subject them to excessive currents would unquestionably be detrimental to the interests of navigation.

Partial regulation of Lakes Michigan-Huron through works located below the head of the river would seem to fit the present need of control at this point. These works should be located in a wide section of the river, perhaps in the vicinity of Stag Island. Their character should be such that a part of the flow could be restrained during the winter months when this restraint should be needed, while not ordinarily reducing the summer flow nor forming any hindrance to navigation. Works of this character would be operated to restore gradually the mean level of Lake Michigan-Huron, and could form a part of more extended works later should it develop that greater control were desirable and that it could be obtained without introducing other evils.

The author expresses the hope that the subject will provoke discussion. As the paper is general, the discussion must be of a similar character. Having in view particularly the needs of navigation, the speaker feels that its require-

ments have been subordinated to those of power. That an increase in prime water is given more weight than better continuous navigation depths. The speaker has attempted to bring out the enormous benefits derived now and in the past from navigation on the Great Lakes, indicating a surplus of $2 000 000 000 now standing to its credit, which is being increased at the rate of $100 000 000 per year, and to show that this navigation should be supplied with the 25-ft. depths it now requires for economical operation.

It has been shown that, whereas the present supply to the lakes is low, there is little to indicate that this condition is permanent; nevertheless, that steps should be taken to prevent the recurrence of extreme low water, but that these steps should be such as not to result in destructive levels during periods of high supply. It has been suggested that the control system proposed might be found disappointing in its final analysis and more expensive than present benefit to power would justify.

Finally, the opinion is expressed that the regulation of Lake Erie should be exhaustively studied, and, that if no insurmountable difficulties appear, it should be put in operation; that minor works should be placed at once in the Lake Michigan-Huron outlet, and that a program for depth betterment to the extent of 25-ft. channels on the Great Lakes should be immediately adopted and entered upon. Such a program would require fifteen years for its consummation. By that time control would have had its preliminary trial, power needs would have increased and presumably would have been supplied either at Niagara or on the St. Lawrence, or both, and during this process, by proper allocation of work on the navigation project, navigation benefits would have been progressively obtained.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

GEORGE HENRY BENZENBERG, Past-President, Am. Soc. C. E.*

DIED MAY 31, 1925.

George Henry Benzenberg was born in New York, N. Y., on May 31, 1847. One year later, his parents removed to Detroit, Mich., where he received his early education. He was graduated as Bachelor of Science in Civil Engineering at the University of Michigan in June, 1867.

Following his graduation, Mr. Benzenberg was engaged for two years on the primary triangulation of the Great Lakes, and from 1870 to 1873, he was employed on the location and construction of the Milwaukee Northern Railroad, from Cedarburg to Green Bay, Wis.

From 1874 to 1899, he served the City of Milwaukee, Wis., as Assistant City Engineer, City Engineer, and President of the Board of Public Works. He also served as Consulting Engineer and, in connection with their water-works or sewerage systems, as a member of several engineering commissions for the following cities: Cleveland, Ohio, Kansas City, Mo., New Orleans, La., Omaha, Nebr., Toledo, Ohio, Racine, Wis., and a number of smaller cities, and completed the water filtration plant of Cincinnati, Ohio.

From 1904 to his death, Mr. Benzenberg served as a Trustee of the Northwestern Mutual Life Insurance Company. As Chairman of the Building Committee of the Trustees, he supervised the building of the home of this Company in Milwaukee, which is the outstanding building in that city and cost about $3 000 000. In 1910, he retired from all professional activities, except those connected with the supervision of the construction of the Insurance Company's building.

With the exception of the first six years, Mr. Benzenberg devoted his professional career to municipal work, many monuments of which indicate his outstanding ability as a resourceful engineer.

While building the water intake tunnel under Lake Michigan, a distance of 6 000 ft., for the City of Milwaukee, numerous blow-outs caused its temporary abandonment. The work was finally completed successfully under Mr. Benzenberg's direct charge.

After the intake water tunnel, crib, and shaft for the water-works of the City of Cleveland had been lost, he took the work under his immediate direction, and recovered them.

His plan for keeping the waters of the Milwaukee and Kinnickinnic Rivers free of sewage pollution by means of fresh lake water pumped into them near the city limits, was a bold undertaking and was condemned by many prominent

* Memoir prepared by the following Committee: John W. Alvord, M. Am. Soc. C. E., *Chairman*, C. F. Loweth, Past-President, Am. Soc. C. E., and T. Chalkley Hatton, M. Am. Soc. C. E.

engineers as impracticable. Mr. Benzenberg built the works and they have been eminently successful. To no other man is the City of Milwaukee so much indebted for its excellent water, sewer, and street improvements, all of which he originally planned, no material changes in these plans having since been found necessary.

In 1911, the University of Wisconsin awarded him the degree of Doctor of Science. In 1893 and 1894, he was President of the American Water Works Association, and, in 1895, became the first President of the American Society of Municipal Improvements.

In addition to his very active professional work, Mr. Benzenberg was quite as active in Masonic circles in which he was widely known throughout his own State and the country. He was a member of all the Masonic organizations, in many of which he had served in official capacities. He received his Thirty-third Degree in 1886 and, at the time of his death, was one of the three active members of the Supreme Council from Wisconsin.

On December 16, 1879, Mr. Benzenberg was married to Allie Wolfrum who survives him; his daughter, Mrs. May Benzenberg Mayer, of New York, N. Y., also survives him.

Throughout his life, Mr. Benzenberg was one of those engineers who, having broadness of vision, bold initiative, and high ideals, commanded the respect and confidence of the community in the improvement of which he took such an active part, and of those with whom he came in contact.

Mr. Benzenberg was elected a Member of the American Society of Civil Engineers on May 2, 1883. He served as Director of the Society from 1895 to 1897; as Vice-President in 1901-02; and as President in 1907.

JOACHIM GOTSCHE GIAVER, M. Am. Soc. C. E.*

DIED MAY 29, 1925.

Joachim Gotsche Giaver, the son of Jens Holmboe Giaver and Hanna Birgit Holmboe, was born on August 15, 1856, in Gjovik, a small town owned by his family, near Tromso, Norway. His father, of a prominent Norwegian family, was a large landholder and one of the leading figures in the fishing industry in Northern Norway. Tutored at home, Mr. Giaver entered the Trondhjems Technical College, at Trondhjem, Norway, from which he was graduated in 1881 with the degree of Civil Engineer.

As the engineering field in Norway was limited at that time he came to America in 1882, and found employment as Draftsman for the Northern Pacific Railroad Company at St. Paul, Minn. At the close of 1883, Mr. Giaver left for Pittsburgh, Pa., to become a Draftsman for the Shiffler Bridge Company and from 1885 to 1890 served as Chief Engineer of that Company. During that period he designed several of the bridges over the Allegheny and Monongahela Rivers at Pittsburgh, and by odd coincidence one of his first

* Memoir prepared by Amasa C. Bull, Esq., Chicago, Ill.

engagements was the design of the structural framework for the Statue of Liberty in New York Harbor.

Mr. Giaver forged rapidly to the front as one of the leading structural designers of the United States and, in 1891, he went to Chicago, Ill., to become Assistant Chief Engineer of the famous World's Columbian Exposition. Having been one of the first to recognize the importance of wind bracing in the structural framework of buildings, he was in charge of perfecting of this feature in the structural design of the buildings for the Exposition. He designed the three-hinge arch in the dome of the Liberal Arts Building which at that time was the largest truss of its kind in the world, having a span of 368 ft. 0 in.

Following the Exposition, Mr. Giaver was engaged in the general contracting business from 1893 to 1896 and served as Bridge Designer for the Sanitary District of Chicago from 1896 to 1898. In this latter position he designed the various bridges over the Chicago Drainage Canal.

During the World's Columbian Exposition, Mr. Giaver had become acquainted with and worked with Chicago's famous architect, the late Daniel H. Burnham, and, in 1898, he became Chief Engineer for the firm of D. H. Burnham and Company, which position he held for 18 years, until 1915. During this time, Mr. Giaver's engineering work was in the practical development of the modern skyscraper from the old spread-footing foundation, cast-iron column, wrought-iron structural framework to the modern caisson foundation and structural steel framework.

As Chief Engineer for the Burnham Company, he had charge of more than four hundred of the largest buildings in the United States, among which were the Flat Iron, Gimbel, Maiden Lane, and Equitable Buildings in New York, N. Y.; the Field Museum, Continental and Commercial National Bank, Railway Exchange, Peoples Gas, Conway, Field, and other buildings in Chicago, Ill.; the Union Station and Post Office in Washington, D. C.; the Frick, Oliver, Smithfield, and First National Bank in Pittsburgh, Pa.; the May Company in Cleveland, Ohio; and the Wanamaker and Land Title Buildings in Philadelphia, Pa. One of his most interesting designs was that for the dome of the Mount Wilson Observatory in California.

In 1915, Mr. Giaver interested himself in the question of licensing structural engineers in the State of Illinois. He was the leader in securing the passage in 1915 of a bill to this effect by the Illinois Legislature, as the result of which structural engineers are allowed to practice their profession on equal terms with the architects in that State. Prior to this it had been impossible for engineers to practice in building operations, except as employees of an architect, as under the former law, building plans could not be lawfully approved unless bearing the signature of a licensed architect.

In 1920, Mr. Giaver was decorated with the order of Knight of St. Olaf, 1st Class, by King Haakon of Norway, in recognition of his prominence as an engineer and his activities on behalf of Norwegians in the United States. This is the highest honor accorded by the Norwegian Government to a non-citizen of that country, Mr. Giaver having received his final papers as a citizen of the United States in 1896.

After the death of Mr. Burnham in 1915, Mr. Giaver established offices in Chicago as a Consulting Engineer, and, in 1916, organized the architectural and engineering firm of Giaver and Dinkelberg. His last work as a member of this firm, finished only a few months before his death, was the engineering design of the 42-story Jewelers Building being erected at Wabash Avenue and Wacker Drive, Chicago, which will be the tallest office building west of New York.

Mr. Giaver was married in New York on September 3, 1883, to Louise C. Schmedling, of Trondhjem, Norway. He is survived by his wife, two daughters, Astrid (Mrs. Ralph Holmboe), Birgit (Mrs. A. C. Bull), and three sons, Erling, Finn J., and Einar W. Giaver.

In private life, Mr. Giaver was an enthusiastic yachtsman. It was his principal hobby and he won many prizes racing his boats on Lake Michigan. He was a Trustee of the Norwegian American Hospital in Chicago; President of the Structural Engineers Association; former Director of the Western Society of Engineers; President of the Norwegian Engineers Association; Past-President of the Chicago Norske Klub; former Vice-Commodore of the Columbia Yacht Club; and a member of the Engineers Club, Svenska Klubben, Chicago Athletic Club, and Chicago Yacht Club.

Mr. Giaver was elected a Member of the American Society of Civil Engineers on September 12, 1916.

CLIFFORD MILBURN HOLLAND, M. Am. Soc. C. E.*

DIED OCTOBER 27, 1924.

Clifford Milburn Holland was born at Somerset, Mass., on March 13, 1883. He was the only child of Edward John Holland and Lydia Frances Hood. His father's ancestors lived in Maine, where they had moved from Massachusetts soon after the Revolution. On his father's side he was descended from Dr. Francis Le Baron, the "nameless nobleman" of early Plymouth. His mother's family came from about Providence, R. I., and Fall River, Mass., and was descended from Roger Williams and others of the earliest settlers.

He attended public schools in Somerset until 1892, then six years in St. Joseph, Mich., and then one year at the High School in Fall River, after which his family moved to Cambridge, Mass. He was graduated from the Cambridge Latin School in 1902 and entered Harvard University, receiving the degree of Bachelor of Arts in 1905 and the degree of Bachelor of Science in Civil Engineering in 1906. It is noteworthy that he completed the work for both degrees in four years.

Before leaving Harvard, he had successfully passed civil service examinations and had been appointed Assistant Engineer in the Engineering Forces of the Rapid Transit Railroad Commission of New York, N. Y. In June, 1906, he began his engineering career with an assignment to the construction

* Memoir prepared by the following Committee: Robert Ridgway, President, Am. Soc. C. E., *Chairman*, and Charles Gilman and Frederick C. Noble, Members, Am. Soc. C. E.

of the Battery-Joralemon Street Tunnel, in which position he continued until its completion in 1908. The work was of an especially difficult and trying character, but he met every task with enthusiasm and liking and performed it very quickly and completely. He had decided to specialize in compressed-air tunneling, but no one could foresee that six years of political delay would elapse before the start of new river tunnels. He was engaged at first in completing the construction records in connection with extra claims, then, for a time, in field charge of a portion of the Fourth Avenue Subway in Brooklyn, N. Y., and, later, the new tunnels coming into prospect, he was assigned to a study of the sections to be recommended. For that use he gathered exhaustive data as to the dimensions, depths, and construction conditions of all similar tunnels that had been built in America and Europe, made a comparative analysis of their stresses and parts, and compiled the results. The work was admirably done and proved most useful, besides giving Mr. Holland a breadth of recorded information on the subject few could equal.

When, in 1914, contracts were let for the Old Slip-Clark Street and Whitehall Street-Montague Street Tunnels, Assistant Engineer Holland was entrusted with direct charge. His age was then thirty-one, but there was no question as to his fitness. His responsibilities were afterward increased when contracts were let for the Willoughby Street, Fourteenth Street, and Sixtieth Street Tunnels, making the contract value of work under his charge about $26 000 000. With five double-tube tunnels, of which four were subaqueous, in progress at once, involving a vast amount of engineering and official detail; with frequent emergencies requiring him to visit widely separated points and render quick, sure decisions at any time of the day or night, the situation would have dismayed any but a man of great courage, ambition, and exceptional ability. In 1916 he was accorded the long desired title and emoluments of Division Engineer. The entire work was carried along according to the best engineering precedents and without serious mishap. At the end of this connection, in June, 1919, the amount and range of Mr. Holland's subaqueous tunneling experience and his close relation to it, as well as his marked success in such work, placed him uniquely in his chosen specialty.

The New York State and New Jersey Interstate Bridge and Tunnel Commissions having been empowered by their respective State Legislatures to cooperate in the construction of a vehicular tunnel under the Hudson River, and realizing, as the Commissioners express it in their Annual Report, "that it was of vital importance to the project to secure the highest degree of technical guidance to develop and carry out this great engineering project", appointed Mr. Holland as their Chief Engineer, which office he assumed July 1, 1919. The report further says:

"After a most careful deliberation the Commission was unanimous in inviting Mr. Clifford M. Holland of Brooklyn, New York, to accept its appointment as chief engineer. The New Jersey Interstate Bridge and Tunnel Commission participated in this action.

"In securing Mr. Holland's acceptance of this position, the Commission felt that it had been singularly fortunate. At the time of his appointment he was tunnel engineer of the Public Service Commission, First District, State of New York, and was in immediate charge of the construction of all the subway

tunnels under the East River. Mr. Holland has probably had a greater and more successful experience in the work of subaqueous tunnel construction than any other member of his profession."

The problem Mr. Holland was given to solve involved a number of features without precise precedent and the expenditure of about $42 000 000. An effective engineering organization had to be built up of men selected for their technical and personal qualifications rather than for political reasons, and it is greatly to the credit of the Commissioners that this right was accorded. Many plans for construction and traffic handling had been proposed and these were carefully studied and investigated. The plan finally recommended by Mr. Holland will provide a pair of cast-iron tubes, shield-driven, each measuring 29 ft. 6 in. in outside diameter and accommodating two lines of traffic in the same direction in each tube. The adoption of this plan, which is now being vindicated, cost the Chief Engineer a severe struggle in which he was obliged to defend his recommendation, with all the grit and resources at his command, against the criticisms and rival plans brought forward by other engineers.

To the question of ventilation of the tunnel after completion Mr. Holland gave a great deal of personal attention, carrying out experiments in conjunction with the University of Illinois, Yale University, and the United States Bureau of Mines, and making frequent trips for that purpose, as well as an examination of ventilation conditions in European tunnels for vehicular traffic. These investigations and others relating to design, power requirements, probable traffic and operating income, and other features, are fully described in his Annual Reports, published by the Commissions, and form an unusually fine record.

While Mr. Holland appeared to be generally in vigorous health, it is probable that he suffered acutely at intervals during several years. His sudden, unexpected death occurred on October 27, 1924, at Battle Creek, Mich., where he had gone for rest and treatment.

Mr. Holland was married on November 5, 1908, to Anna Coolidge Davenport, of Watertown, Mass., who survives him, as do four children, Anne, Clarissa, Benita, and Lydia.

At the time of his death Mr. Holland was a member of the Board of Direction, which adopted the following resolution:

"On October twenty-seventh, nineteen hundred and twenty-four, Clifford Milburn Holland, obedient to the call of the Divine Providence, entered the great Beyond. His friends and associates cherished his memory. He was strong, active, and fearless in the right, kind and considerate of all, a man of attainment, of initiative, and of native ability. Young in years, but old in accomplishment, he lived intensively, yet the joy of living was always his.

"Few, indeed, are endowed with the qualities of leadership, with the ability which inspires confidence and with the inborn knowledge of men and things as was he. His future lay great before him. His past was one of inspiration. His work stands as an enduring monument.

"We, his fellow members on the Board of Direction of the American Society of Civil Engineers assembled in meeting at Pittsburgh, Pennsylvania, on this first day of December, nineteen hundred and twenty-four, desire to record our sorrow which is beyond the power of words to express. The

Society and the profession have sustained an incalculable and inexpressible loss.

"In mute and reverent silence we stand in the presence of an inscrutable Providence. Yet in a measure we find our consolation in being the better and the richer because in the fullness of his life we knew him, and knowing, did love and honor him.

"Our sympathy goes out to his family and to all he held dear and it is the desire of this Board of Direction, speaking for all the membership of the Society, that this expression be entered upon its minutes, be printed in its *Proceedings* and be, in suitable form, sent to the family of Clifford Milburn Holland."

He was active in the affairs of the Harvard Club of New York and the Harvard Engineering Society. Following is the minute passed by the Board of Managers of the Harvard Club:

"In the death of Clifford Milburn Holland, the Harvard Club of New York has lost a loyal and earnest member. He took a deep interest in the work of the Club, serving faithfully as a member of the Committee on Admissions and as a member of the Board of Managers. He was prominent in the affairs of the Harvard Engineering Society during the past twelve years, in which time he held all of its offices. He was equally interested in the new Harvard Engineering School, a member of its Visiting Committee, and an inspiration to the undergraduate studying engineering.

"He fought squarely and with all the energy at his command for whatever he deemed to be right. A loyal and unselfish friend he was always ready to help those who needed his assistance. Forceful and diplomatic, the benefit of his counsel was sought by all. Acknowledged to be an authority in that branch of engineering to which he had devoted his working life, his career was cut short at the moment of bringing to fruition the monumental work that bears his name.

"In grateful recognition of his services to Harvard and to his fellow men, we—the Board of Managers of the Harvard Club—spread upon our records this minute to his memory."

At a meeting of the Harvard Engineering Society, held for memorial exercises, action was taken to rename the engineering scholarship stipend established at Harvard by that Society, to be known thereafter as the Clifford M. Holland Memorial Aid in Engineering. A resolution was adopted, from which is excerpted the following:

"In the death of Clifford Milburn Holland of the Class of 1906, The Harvard Engineering Society has lost one of its most active and useful members, loved and respected by all. He served the Harvard Engineering Society of New York as Treasurer in 1914-15, Secretary in 1915-19, and Vice-President in 1919-20, and The Harvard Engineering Society as Second Vice-President, 1920-21, and as President, 1922-23, in all of which positions he left a distinguished record. He gave unstintingly of his time and strength in the discharge of his duties and his support and counsel were ever available in the conduct of the affairs of the Society. He took a very active part in the consolidation of the Association of Harvard Engineers, the Lawrence Scientific School Association, and the Harvard Engineering Society of New York, which resulted in the present strong National Society. He took a great interest in the new Harvard Engineering School and, as Chairman of the Committee on University Relations of the Society, directed the influence of the alumni body in its co-operation with and assistance to the School."

Mr. Holland was also a member of the American Association of Engineers and of the Engineers' Club of New York.

As a tribute to the Chief Engineer who gave so much to make the tunnel a success, and as a lasting monument to his memory, the New York State and the New Jersey Interstate Bridge and Tunnel Commissions, within a short time after his death, officially named the tunnel "The Holland Tunnel". This fitting and magnanimous act is deeply appreciated by Mr. Holland's associates and doubtless will be by engineers generally.

Engineering work with which he was entrusted was always managed and executed according to the best standards, that result being secured in a manner that won him the regard, as well as the respect, of the contractors who executed the work, as witnessed by the following tribute:

"The Executive Council of the General Contractors' Association desires to place on record, as a minute of this meeting, its expression of sorrow over the death of Clifford M. Holland, Chief Engineer of the Hudson Vehicular Tunnel.

"Mr. Holland possessed with his extraordinary ability as an engineer those qualities of tact, courtesy and fairness that won the respect and affection of all those with whom he came in contact, socially or professionally.

"In his death the community has lost a public servant of inestimable value; the engineering profession, a brilliant leader; his intimates and associates, a beloved friend.

"As contractors, many of whom were privileged to come in contact with him, we honor ourselves in paying this tribute to Clifford M. Holland, who combined unswerving loyalty to his professional duty, with that spirit of honesty that conceded even-handed justice in all his dealings."

Resolutions of sorrow and appreciation were adopted by the New York State and New Jersey Interstate Bridge and Tunnel Commissions, and by the Board of Transportation of New York City which had succeeded to the control of the rapid transit work with which Mr. Holland was formerly connected.

It was characteristic of him that, even when his professional burdens were the heaviest, he found time to serve on committees of the Society and to share generously in its counsels. His earnest sympathies and activities were always to be enlisted in any movement he felt to be for the real benefit of engineers and the profession.

The loss of Clifford Holland was felt inconsolably by his many sincere friends, and to a peculiarly notable degree even by those whose acquaintanceship was slight. His rare personal qualities, so manifestly genuine, attracted friendship at once. In any gathering, his presence would be a buoyant, lively influence, or his absence would be noted and regretted. In every way he was human, sociable, likable. His place in many hearts will hardly be filled.

The following verse is a tribute *in memoriam* by Mrs. Marguerite Janvrin Adams, the wife of one of Mr. Holland's classmates:

"He is not dead who leaves on earth enshrined
His image in a work of faith and skill;
He has not died, when we can listen still
To words of praise that everywhere we find
On comrades' lips. Each day will subtly bind
Us closer to him, whose untiring will
Struggled thus valiantly and nobly 'til
The task completed lay,—to aid mankind.

He who has made on earth one work complete
To live beyond this little span of days
That we call life—at parting of the ways
Has left a monument before our feet
At which we bow in reverence. Thus we stand
So near in homage we can clasp his hand."

Mr. Holland was elected an Associate Member of the American Society of Civil Engineers on October 29, 1912, a Member on April 18, 1917, and served as a Director from 1922 until his death.

CHARLES WALTER LAWRENCE, M. Am. Soc. C. E.*

DIED APRIL 1, 1925.

Charles Walter Lawrence, the son of C. F. and Mary A. Lawrence, of Claremont, Calif., was born at Plain Grove, Lawrence County, Pa., on June 24, 1872.

He received his engineering education at Pennsylvania State College, from which he was graduated in 1897, with the degree of Bachelor of Science in Civil Engineering. He returned to his Alma Mater in 1904 to receive the degree of Civil Engineer.

In September, 1897, Mr. Lawrence was appointed Instructor in Civil Engineering at Pennsylvania State College, where he remained until June, 1899, when he accepted a position as Draftsman with the Pennsylvania Steel Company, at Steelton, Pa.

In October, 1901, he entered the employ of the Brown-Ketchum Iron Works, at Indianapolis, Ind., as Structural Engineer on the design of steel building and bridge work. In September, 1904, he returned to Pennsylvania State College as Assistant Professor of Structural Engineering, which position he retained until September, 1906, when he became Professor of Civil Engineering at the James Milliken University, Decatur, Ill., which Department he established and directed until June, 1909. His summer vacations from 1906 to 1909 were spent in practical work as Engineer with the Brown-Ketchum Iron Works, on steel design, as Designing Engineer with the Westlake Construction Company, St. Louis, Mo., and as Engineer with the American Bridge Company, Lassig Plant, on design and checking of details.

For fourteen years prior to his death, Professor Lawrence served as Professor of Civil Engineering and as Dean of that Department at the University of Southern California, Los Angeles, Calif. It was largely through his efforts that the Department was developed into one of the leading engineering colleges in the United States.

In Professor Lawrence's early demise, the Engineering Profession loses an able and clear-headed thinker, and the community and the educational institution which he served, an honest, public-spirited man, and an able teacher.

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* Memoir prepared by Henry Z. Osborne, M. Am. Soc. C. E.

To those of his many friends who came in contact with him, the outstanding features of his character were his whole-souled and unselfish devotion to his profession, his loyalty and friendship for those with whom he was associated, simplicity of character, and forgetfulness of self.

Professor Lawrence was a Charter Member and Vice-President of the Los Angeles Chapter of the American Association of Engineers, later becoming a member of its Board of Directors; a member of the Society for the Promotion of Engineering Education; and a member of Chi Epsilon, the National Honorary Civil Engineering Fraternity.

He is survived by his widow and two sons, Harold Glover and Charles Oliver Lawrence.

Professor Lawrence was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908, and a Member on January 20, 1922.

PAUL HOWES NORCROSS, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Paul Howes Norcross was born in Atlanta, Ga., on January 19, 1880, the eldest son of Dr. Virgil C. and Fanny Howes Norcross, and a grandson of Jonathan Norcross, the first Mayor of Atlanta and one of its pioneer citizens.

After his preparatory education in the schools of Atlanta, Mr. Norcross entered the Georgia School of Technology and was graduated, in 1902, with the degree of Textile Engineer. Immediately afterward, he spent about six months in one of the large Southern cotton mills, but did not find this branch of engineering especially attractive to him. Accordingly, he obtained a position with the Carolina Bridge Company, of Burlington, N. C., first in the Drafting and Designing Department, but because of his sales ability, which was even then apparent, he was soon transferred to the Contract Department and, later, made Special Sales Representative, in charge of territory covering eleven of the Southern States.

The Carolina Bridge Company was later merged into the Virginia Bridge and Iron Company and Mr. Norcross continued with this organization, having been most successful in the promotion and sale of a large number of highway and railway bridges throughout the South. In 1905, he accepted a position with the Westinghouse, Church, Kerr and Company, in connection with the structural design of the Pennsylvania Station in New York, N. Y. After the completion of this work, he became a member of the Staff of Herbert C. Keith, M. Am. Soc. C. E., Consulting Structural Engineer, on special bridge design for the New York Central Railroad, in the electrified zone between Harmon, N. Y., and the Grand Central Station, New York.

In 1906, Mr. Norcross formed a partnership with Gabriel R. Solomon, M. Am. Soc. C. E., under the firm name of Solomon-Norcross Company, with principal offices in Atlanta. The firm engaged in general consulting practice

* Memoir prepared by George W. Fuller, Frederick H. McDonald, and Gabriel R. Solomon, Members, Am. Soc. C. E.

throughout the South, covering all classes of municipal and hydraulic work. In 1910, a branch office was opened in Watervliet, N. Y., and Francis J. Keis, M. Am. Soc. C. E., was taken into the partnership, under the firm name of Solomon, Norcross and Keis, on Northern work. Mr. Solomon retired from the organization, in 1918, on being commissioned an officer in the Army, and the firm operated under the name of Norcross and Keis until January 1, 1923, when Mr. Keis retired. Mr. Norcross continued the Atlanta Office, as a Consulting Engineer, until the time of his death.

Mr. Norcross' engineering activities, during the last few years, were largely concentrated on water supply and filtration projects, his firm having served more than fifty municipalities on this particular type of work. At the time of his death, he was Consulting Engineer to the City of Atlanta, where the $3 500 000 pumping and filtration plant was nearing completion, and was in charge of the design and supervision of the new $1 300 000 water supply for Spartanburg, S. C.

Among the many projects designed and supervised by Mr. Norcross and his partners might be mentioned: Water supplies at Prattville and Talladega, Ala.; DeFuniak Springs, Milton, Haines City, and Sebring, Fla.; Cartersville, Rome, Jefferson, and Madison, Ga.; Amory, Miss.; Watervliet, N. Y.; Aiken, S. C.; and Cleveland and Morristown, Tenn.; also, special work for the Island of Bermuda; sewage disposal plants at Lexington, Ky., and a number of other cities; water power reports for the Broad River Power Company, Alabama Electric Company, and Selma Electric Company; and flood-control works at Watervliet, N. Y.

At the outbreak of the World War, Mr. Norcross was appointed as Supervising Engineer at Camp Jackson, Columbia, S. C., and, on the completion of this work, served in a similar capacity at Camp Humphreys, Accotink, Va. He was later made Acquisition Agent for Camp Benning, Georgia, and purchased approximately 30 000 acres of land for the Government. He was then called to Washington, D. C., and became a member of the Civilian Staff of the Water Supply Branch of the Construction Division of the Army, in charge of Dabney H. Maury, M. Am. Soc. C. E., then Lieutenant-Colonel, and while on this work, was given many assignments in connection with the water supplies of the camps, cantonments, and Regular Army posts. After the Armistice, Mr. Norcross returned to Atlanta and, with Mr. Keis, continued his consulting practice.

Among his last professional engagements was that as a member of a group of twenty-eight engineers constituting a Board of Review to pass on numerous aspects of the problems of the Sanitary District of Chicago, Ill. He served as Secretary of the Board and as a member of its Committees on Water Supply and on Water Power. The unanimity and promptness with which the Board reported its findings and conclusions on this many-sided proposition of unusual local, interstate, National, and international significance was due in no small measure to the organizing ability, tactfulness, and energy of Mr. Norcross.

Mr. Norcross' activities outside his engineering office were numerous and varied. He had unusual social talents, was thoroughly at home in any company, and was an exceptionally interesting and ready speaker. His keen wit caused

him to be sought on all occasions as a presiding officer, and his appropriate remarks greatly enlivened every meeting at which he was present. No one could place a body of men more quickly at their ease, nor overcome a formal reserve more easily, than he.

For many years he was active in the Atlanta Rotary Club and served the Club as President. In 1924, he had the distinction, unique for an engineer, of being made President of the Atlanta Chamber of Commerce, having served that organization as Director, and as Chairman of a number of important committees. He was also President of the Alumni Association of the Georgia School of Technology.

Mr. Norcross lost his life, with twenty-two others, when the steamer *H. E. Norman,* carrying delegates of the First Annual Convention of the Association of Engineers of the Mid-South, sank in the Mississippi River south of Memphis, Tenn., near Kohoma Landing. His attendance at the Convention was in performance of his duty as a Director of the Society in aiding the consideration of the formation of a Local Section. His last speech was on the ill-fated *Norman,* in the course of a river inspection trip forming part of the Convention program, during which he urged that the Engineering Profession should organize so as to be of as much aid as practicable to young engineers.

He was a Scottish Rite Mason, a Shriner, and an Elder in the First Presbyterian Church of Atlanta.

Mr. Norcross was married, in November, 1907, to Mattie L. Holt, of Graham, N. C., who died in October, 1920, leaving a daughter, Catherine Holt Norcross, who survives her parents.

His many friends among the engineers of the United States were particularly gratified at the appreciation of his worth as a citizen shown at Atlanta on May 31, 1925, by a Community Memorial Service, when leading public officials, representatives of important associations, and of his church, characterized him as:

"A Dutiful Son—A Devoted Husband
A Loving Father—A Loyal Friend
A Public-Spirited Citizen
A True Christian".

Mr. Norcross was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906 and a Member on May 7, 1913. At the time of his death he was serving as a Director of the Society to which office he had been elected on January 21, 1925.

WILLIAM ROTCH, M. Am. Soc. C. E.*

DIED AUGUST 14, 1925.

William Rotch was born at New Bedford, Mass., on July 22, 1844, the eldest son of William J. and Emily (Morgan) Rotch. On both sides of his family, he came from distinguished ancestry. His great-great-great-grandfather, Joseph Rotch, already a successful shipowner, came to New Bedford from

* Memoir prepared by Albert B. Drake, M. Am. Soc. C. E.

Nantucket, Mass., in 1765. Joseph Rotch and his sons, William and Francis, found New Bedford a farming town doing a little whaling off the coast, and by their energy they established the industry that made the town the great whaling center of the world. Francis Rotch built and owned the *Dartmouth,* the best known of the three ships of the Boston "Tea Party". William Rotch (1738 to 1828) owned the ship, *Bedford,* which first displayed the American flag in foreign waters in 1783. After the Revolution, he established the whale fishery in France, but returned to the United States following the French Revolution. With the decline of the whaling industry, the Rotchs turned their energies to cotton manufacturing. At various times Mr. Rotch's father, William J. Rotch, was President and Director of most of the older New Bedford cotton mills. On his mother's side, Mr. Rotch was a grandson of Charles W. Morgan, one of the great whaling merchants of New Bedford, whose whaling vessel, the *Charles W. Morgan,* has been preserved as a memorial to the whaling industry.

William Rotch was educated at the Friends Academy of New Bedford, and was graduated from Harvard University in 1865, with the degree of A. B. He then entered the Ecole Centrale des Arts et Manufactures, in Paris, France, from which he was graduated in 1869, ninth in a class of 225, with the degree of "Ingenieur Civil".

Mr. Rotch began his professional career in 1871 as Assistant Engineer of the Fall River, Mass., Water-Works, then in course of construction, and served as Chief Engineer from 1874 to 1880, when the works were completed. From 1880 to 1882, he was Consulting Engineer and Purchasing Agent of the Mexican Central Railway Company, the Sonora Railway Company, the Atlantic and Pacific Railroad Company and the California Southern Railroad Company, the lines of the last two now forming the western portion of the main line of the Santa Fé System. From 1884 to 1891, he was Consulting Engineer of the Cleveland, Canton, and Southern Railroad Company, the Lakeside and Marblehead Railroad Company, the New Bedford and Fall River Railroad Company, and other railroad companies in Ohio and Massachusetts.

In 1881, Governor Long of Massachusetts appointed Mr. Rotch as Engineer and Member of the Commission which finally established the boundary line between Massachusetts and Rhode Island that had been in dispute for more than 250 years. He served on this Commission until 1883.

In 1894 and 1895 he was a member of the Re-Organization Committee of the Atchison, Topeka, and Santa Fé Railway, and from 1895 to 1900, he served as a Director of that road. He was also a Director of the Mexican Central Railway Company from 1880 to 1900, and had charge of the first train which ran from Boston to the City of Mexico, in April, 1884. From 1886 to 1894, Mr. Rotch was President of the Tremont Electric Lighting Company and the Consolidated Electric Manufacturing Company, and a Director of its successor, the Walker Company, which Company installed the complete electric plant in the Waldorf-Astoria Hotel in New York, N. Y., then the finest electric lighting plant in America.

Mr. Rotch was an official in many Massachusetts corporations. From 1913 to 1922, he was President of the Federal Wharf and Storage Company; from 1918 to 1922, Vice-President of the State Wharf and Storage Company; and from 1918 to 1922, President of the Fibre Manufacturing Company. He was also President of the Railroad Wharf and Storage Company, the Denbigh Mining Corporation, the Terrible Dunderberg Mining and Powder Company; Vice-President of the Bonanza Development Company; Treasurer of the Broadway Storage Company; Trustee and Treasurer of the Nonquitt Real Estate Trust; Managing Director of the Rotch Travelling Scholarship for Architects; and a Director of the Infants Hospital and the Adams Nervine Hospital. In 1870, he was elected the first President of the Union for Good Works in New Bedford.

Mr. Rotch was President of the Alliance Francaise of Boston and Cambridge, Mass.; Governor of the Massachusetts Society of Colonial Wars; an Honorary Member of the Boston Society of Architects; a member of the Association des Anciens Eleves de l'Ecole Centrale of Paris, the Bostonian Society, the Somerset and Harvard Clubs of Boston, and the Wamsutta Club of New Bedford. He was also a member of the Arlington Street (Unitarian) Church, of Boston.

He was married in 1873 to Mary Rotch Eliot, who survives him, together with three children, Edith Eliot, Charles Morgan, and Clara, the wife of Dr. Channing Frothingham.

Mr. Rotch had both his office and residence in Boston for many years. In spite of his eighty-one years, he had been in excellent health and had attended the Harvard-Yale Baseball Game in June, 1925, at which he was the oldest graduate present. Taken ill at his summer home at Nonquitt, Mass., he was taken to the hospital at New Bedford on June 25, where following an operation his condition was thought to be improving until he suffered a heart attack which caused his death. Possessed of a wonderfully genial personality, Mr. Rotch leaves a host of friends.

Mr. Rotch was connected with the American Society of Civil Engineers for fifty-two years, having been elected a Member on March 5, 1873.

JOSEPH THOMPSON STUART, M. Am. Soc. C. E.*

DIED FEBRUARY 13, 1925.

Joseph Thompson Stuart was born at Port Royal, Pa., on May 28, 1858, and received his early education at the public schools in Allegheny, Pa.

On August 4, 1881, he entered the employ of the Pennsylvania Railroad Company (Pittsburgh Division), as Chainman, but, in October, was transferred to the Construction Department.

In 1886, Mr. Stuart was promoted to be Assistant Engineer of Construction, in charge of change of line and the construction of a four-track arch

* Memoir compiled from information furnished by Joseph J. Vogdes, M. Am. Soc. C. E.

bridge at Brinton, Pa., on the Pittsburgh Division. In 1887, the construction of the 33d Street Bridge and the extension of the train-shed at Pittsburgh, Pa., was effected under his direction.

From 1888 to 1890, Mr. Stuart was engaged in reconstruction work, for the Pennsylvania Railroad Company, building shops and yards and rebuilding line and the South Fork and Viaduct Bridges after the Johnstown flood. He also rebuilt the Portage Tunnel at Gallitzin, Pa.

In 1891, he had charge of the change of line at Principio, Md., and also at Washington, D. C., including the elimination of the grade crossing for the Union Station. During this time he also rebuilt the Ebensburg Branch of the Pennsylvania Railroad from Cresson to Kaylor, Pa.

From 1892 to 1895, Mr. Stuart was in charge of the construction of the Broad Street Station at Philadelphia, Pa., and, in 1895 and 1896, of the Delaware Bridge for the Pennsylvania Railroad at Philadelphia, which included the caisson work and superstructure. During 1897 and 1898, the change of line at Bacon Hill and Northeast, Md., of the Philadelphia, Baltimore, and Washington Railroad, was under his direction. In 1899 and 1900, he built the Bond Street Terminal Warehouse and Pier at Baltimore, Md., and in 1900 and 1901, he constructed the Camden Terminal at Camden, N. J.

Mr. Stuart was in charge of the elevation of the railroad through Chester, Pa., from 1901 until 1903. On August 1, 1903, he was appointed Engineer of Construction in charge of all construction work of the Pennsylvania Railroad Company between Philadelphia and Washington, D. C., under the supervision of the Chief Engineer. This included the building of the Wilmington and Washington Elevated Railroads.

On January 31, 1904, he resigned his position with the Pennsylvania Railroad Company and entered the contracting business with Mr. Andy Brann. This partnership afterward became Brann and Stuart Company, Incorporated, of which Company Mr. Stuart was President at the time of his death which took place at Dunedin, Fla., on February 13, 1925.

He was a member of the Engineers Club and the Union League of Philadelphia, the Scotch-Irish Society of Pennsylvania, the Philadelphia Yacht Club, and an Honorary Member of the Philadelphia Lehigh Club.

Mr. Stuart was elected a Member of the American Society of Civil Engineers on May 4, 1904.